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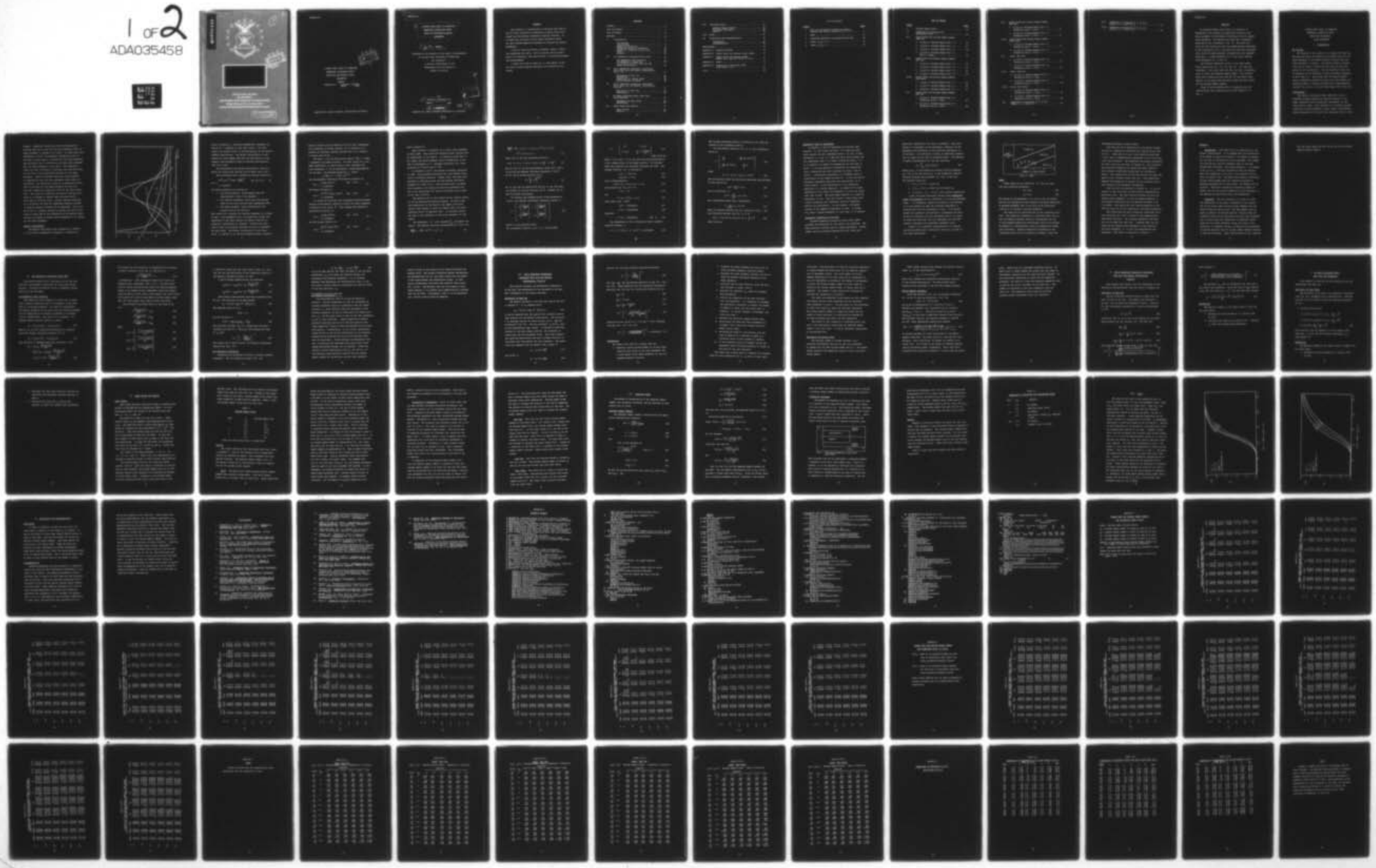

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A MONTE CARLO STUDY OF COMPOSITE SEQUENTIAL LIKELIHOOD RATIO TESTS FOR THE WEIBULL SCALE PARAMETER

THESIS

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Preface

This thesis is a continuation of previous work done at the Air Force Institute of Technology on Monte Carlo techniques with the Weibull probability density function. It is hoped that the work of this thesis provides a basis for more refined adaptive procedures in testing for Weibull parameters.

I wish to thank my advisor, Professor Albert H. Moore for suggesting the topic. Both my advisor and my reader, Major C.W. McNichols, were most helpful in providing guidance and encouragement.

I would also like to thank Dr. H. Leon Harter of the Air Force Flight Dynamics Laboratory for sponsoring this thesis.

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Abstract

An extensive Monte Carlo effort is conducted to sequentially test between two hypotheses concerning the scale parameter of the Weibull distribution with an unknown shape parameter. The methods are based upon a procedure described by Cox in Sankhya A, Vol. 25. The first two tests use test statistics that are asymptotically equivalent to the likelihood ratio. The third test uses the likelihood ratio with the shape parameter estimated and with expanded boundaries. A scale parameter of 1.0 was tested against scale parameters of 1.5 and 2.0.

The location parameter is zero in all three tests. Error bounds were .20, .15, .10, and .05. All tests are uncensored. The three tests are then run with a truncation point of twice the expected sample number. Five hundred Monte Carlo repetitions are used for all the above tests. Results of the tests show the actual alpha and beta errors and the average sample numbers.

Power of the truncated tests is computed using two hundred Monte Carlo repetitions for error bounds of .2 and .1.

A MONTE CARLO STUDY OF COMPOSITE SEQUENTIAL LIKELIHOOD RATIO TESTS FOR THE WEIBULL SCALE PARAMETER

I. Introduction

The Problem

The purpose of this thesis is to examine the Type One and Type Two errors of sequential likelihood tests for the scale parameter of the Weibull density function when the shape parameter is unknown. The general method of sequential testing using a likelihood ratio test has been suggested by Cox and Bartlett. Cox and Bartletts' works are an extension of the sequential testing procedure developed by Wald (Ref 6:5). Wald found that the general method using a sequential probability ratio test could reduce the expected number of required observations to approximately one-half the size of a similar fixed sample test (Ref 26:1).

Significance

The Weibull function has been observed to be an extremely flexible probability function because it can model components having increasing, decreasing, or constant failure rates. This flexibility is achieved through variation of three parameters, scale, shape, and location. These parameters will be more fully explained later in this

chapter. Sequential procedures have been developed by Williams (Ref 28) to test for the scale parameter when the shape parameter is assumed to be known. In cases where this assumption is valid, the sequential probability test of Williams is quite useful in testing for the scale parameter. The difficulty lies in testing for the same scale parameter when the shape parameter is not known. This problem is likely to occur in testing a relatively new item which does not yet have a well-established estimate for the shape parameter. There are methods other than those proposed to sequentially test the scale parameter under these circumstances. One possibility would be to graphically determine an estimate for the shape parameter and then use Williams' sequential probability test for the scale parameter. The reason for examining alternative testing procedures is to attempt to improve upon the power and/or efficiency of existing tests. Another possible advantage is to eliminate the need for cumbersome graphical methods if adequate numerical methods can be developed. This could allow use of a computer routine to conduct the entire test. Figure 1 shows some of the flexibility of the Weibull model and also the danger of using a poor estimate of k in testing for the scale parameter.

Weibull Distribution

The Weibull distribution was proposed by W. Weibull in 1939 without mathematical foundation. According to

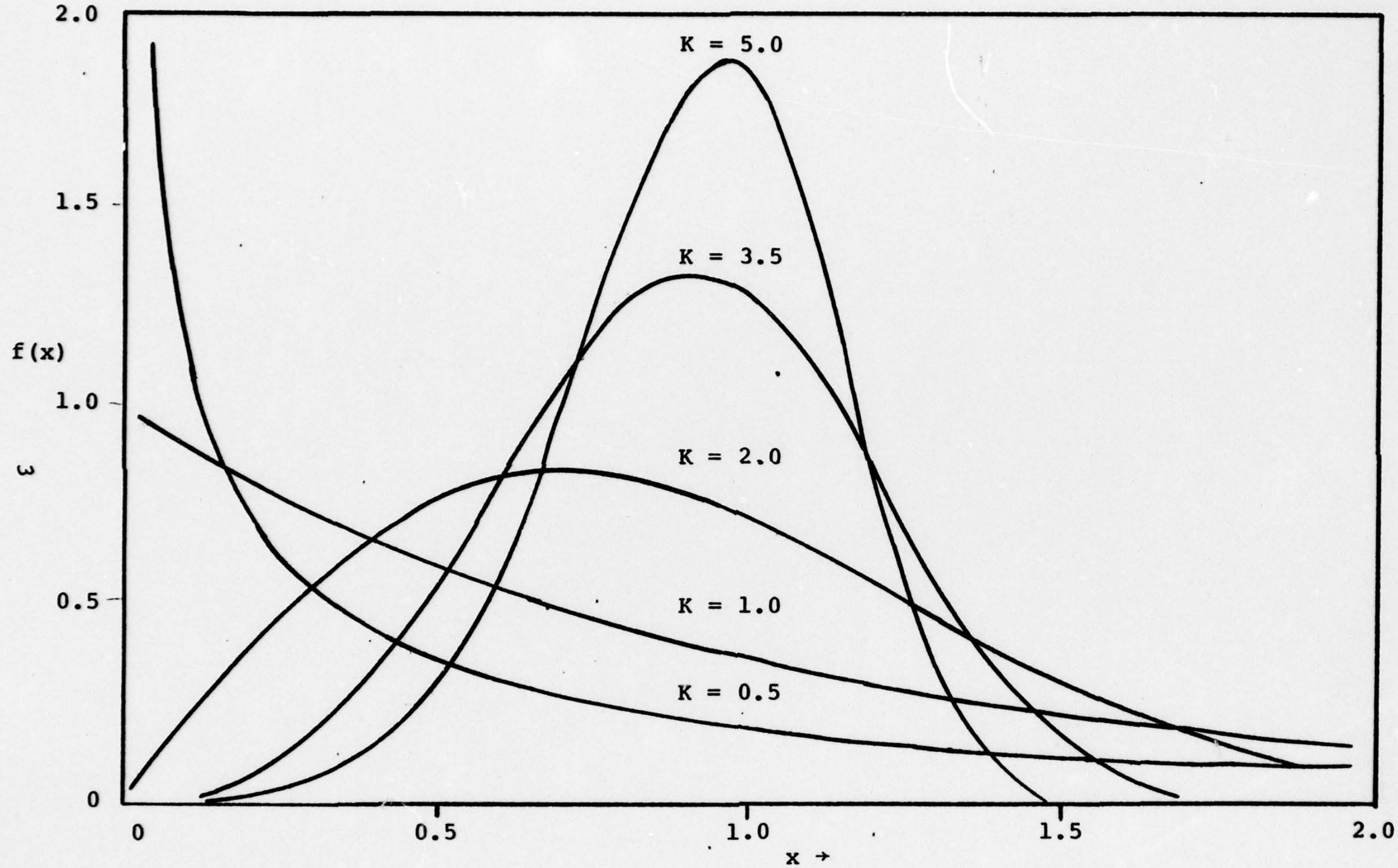

Fig. 1 Plot of the Weibull Probability Density Function for Various Values of K, (θ = 1). (Ref 21:3)

Soviet literature, a "rigorous mathematical treatment was done by B. V. Gnedenko in 1949 [Ref 7:123]." For this reason, the Soviets refer to the distribution as the Weibull-Gnedenko distribution. The Weibull distribution is considered the most complex and also the most popular of the commonly used parametric family of failure distributions (Ref 3:16; 2:46).

The distribution was further publicized in 1949 as a method for testing the fatigue life of metal (Ref 10:14).

The distribution has a probability density function of

$$f(x) = \begin{cases} k\theta^{-k}(x-w)^{k-1} \cdot \exp\left(- \frac{(x-w)}{\theta}\right)^{k} & x \geq w \quad c \geq 0 \quad \theta, k > 0 \\ 0; \text{ elsewhere} \end{cases} \qquad (1)$$

The three parameters are defined as:

θ - the scale parameter, which depends upon the characteristic life of the function.

w - the location parameter, which tells the minimum value of the first failure time (guaranteed life).

k - the shape parameter, which characterizes the failure rate of the function.

This thesis will consider the location parameter to be equal to zero. This assumption will not effectively hinder the usefulness of the study because the Weibull function is primarily a description of lifetime. Allowing the lifetime to start and possibly terminate at zero is reasonable for many cases. The Weibull distribution in this form, (w=0), is referred to as the Two Parameter Weibull function.

Another insight into the meanings of the k and θ parameters with reference to fatigue failure, is to consider k as a function of the mean ultimate strength and θ as a function of stress (Ref 10:32).

The mean, u, of the distribution equals $\theta\Gamma(\frac{1}{k} + 1)$ where Γ represents the gamma function. For most values of k, the gamma coefficient is nearly equal to one, therefore the scale parameter, θ, can be considered a rough approximation for the mean. The standard deviation, σ, equals

$$\theta[\Gamma(\frac{2}{k} + 1) - \Gamma^2(\frac{1}{k} + 1)]^{1/2}$$

The two-parameter cumulative distribution function is:

$$F(x) = \begin{cases} 1-\exp[-(x/\theta)]^k & x \geq 0 \quad k,\theta > 0 \\ 0; \text{ elsewhere} & \end{cases} \qquad (2)$$

The density function is

$$f(x) = \begin{cases} k\theta^{-k}x^{k-1}\cdot\exp[-(x/\theta)^k] & x \geq 0 \quad \theta,k > 0 \\ 0; \text{ elsewhere} & \end{cases} \qquad (3)$$

It should be noted that the literature contains another version of the Weibull function in which the scale parameter is defined as the value, θ^k. This changes the cumulative distribution function to

$$F(x) = \begin{cases} 1-\exp[-(x^k/G)] & x \geq 0 \quad G,k > 0 \\ 0; \text{ elsewhere} & \end{cases} \qquad (4)$$

and the density function to

$$f(x) = \begin{cases} \frac{k}{G}\, x^{k-1}\cdot\exp[-x^k/G] & x \geq 0 \quad G,k > 0 \\ 0; \text{ elsewhere} & \end{cases} \qquad (5)$$

where G equals θ^k.

Some confusion is possible as to which scale parameter is being used. Test statistics employing both versions will be constructed in this thesis. It should be noted that in all cases, the scale parameter under consideration in the test of hypotheses is θ. Parameter estimation will refer primarily to $\hat{\theta}$ and $\hat{k}$ since $\hat{G} = \hat{\theta}^{\hat{k}}$.

To estimate k and θ, the method of maximum likelihood is used. This procedure maximizes the joint density function or likelihood function. Thoman, Bain, and Antle have provided an unbiasing factor for the maximum likelihood estimate of k (Ref 25:11). Petrick shows that, as sample size increases, the unbiasing multiplier approaches one and is not significant for tests with sample size as small as six (Ref 21:28-30).

The methods used in this thesis did not require unbiasing of the maximum likelihood estimates. As can be seen upon observation of the results, the sample sizes were usually large enough to cause the Thomas, Bain, and Antle unbiasing factor to approach unity. No unbiasing factor was used in the estimation of the scale parameter in this thesis.

The expression, "Σ," will represent $\sum_{i=1}^{n}$ throughout the thesis. The maximum likelihood expressions for θ and k are

$$\frac{\partial L(x)}{\partial \theta} = - \frac{nk}{\theta} + k\theta^{-k-1} \Sigma x_i^{\,k} = 0 \qquad (6)$$

$$\frac{\partial L(x)}{\partial k} = \frac{n}{k} - n \ln \theta + \Sigma \ln x_i + \theta^{-k} \ln \theta \, \Sigma \, x_i^{\,k}$$

$$- \theta^{-k} \Sigma x^k \ln x = 0 \qquad (7)$$

where L(x) is the log likelihood function,

$$L(x) = n \ln k - n \ln \theta + (k-1) \ln \left(\frac{x}{\theta}\right) - \Sigma \left(\frac{x}{\theta}\right)^k \qquad (8)$$

Eqs (6) and (7) can be simultaneously solved for θ and k to provide the maximum likelihood estimates, $\hat{\theta}$ and $\hat{k}$. Eq (6) can be solved directly for $\hat{\theta}$.

$$\hat{\theta} = \left[\frac{1}{n} \Sigma x^{\hat{k}}\right]^{1/\hat{k}} \qquad (9)$$

Eq (9) can then be substituted into Eq (7) but this does not produce an analytic solution for $\hat{k}$. Instead, Eq (7) must be solved iteratively.

To evaluate the standard error of the estimates of θ and k, the maximum likelihood information matrix, I,

$$I = \begin{bmatrix} -E\left[\frac{\partial^2 L(x)}{\partial \theta^2}\right] & -E\left[\frac{\partial^2 L(x)}{\partial \theta \partial k}\right] \\ -E\left[\frac{\partial^2 L(x)}{\partial k \partial \theta}\right] & -E\left[\frac{\partial^2 L(x)}{\partial \theta \partial k}\right] \end{bmatrix} \qquad (10)$$

where E is the expected value.

For the Weibull function, f(x; θ, k), this becomes

$$I = \begin{bmatrix} k^2 & \Gamma'(2.0) \\ \Gamma'(2.0) & \frac{1}{k^2} + \frac{\Gamma''(2.0)}{k^2} \end{bmatrix} \qquad (11)$$

(Ref 23:45-46)

where $\Gamma'(2.0)$ and $\Gamma''(2.0)$ are the first and second derivatives, respectively, of the gamma function. To evaluate these two functions the following procedure was used. The digamma function, (ψ), is defined as

$$\psi(a) = \Gamma'(a)/\Gamma(a) \qquad (12)$$

$$\psi(a)\Gamma(a) = \Gamma'(a) \qquad (13)$$

and by differentiation

$$\psi(a)\Gamma'(a) + \psi'(a)\Gamma(a) = \Gamma''(a) \qquad (14)$$

Substituting for $\Gamma(2)$ and $\Gamma'(a)$

$$\Gamma(2) = 1.0 \qquad (15)$$

and

$$\Gamma''(a) = \psi^2(a) + \psi'(a) \qquad (16)$$

From tables (Ref 1:268)

$$\psi(2) = .4227843351 \qquad (17)$$

$$\psi'(2) = .6449340668, \qquad (18)$$

therefore

$$\Gamma''(2) = .8236806601 \qquad \text{(Ref 5)} \qquad (19)$$

The determinant of this information matrix becomes a positive constant, Δ,

$$\Delta = 1 + \Gamma''(2.0) - (\Gamma'(2.0)^2 = 1.644934066 \qquad (20)$$

The variance-covariance matrix is defined as 1/n times the inverse of the information matrix.

For the Weibull function, f(x; G, k), the information matrix is

$$I = \begin{bmatrix} \frac{1}{G^2} & -\frac{1}{Gk}(\ln G+\psi(2)) \\ -\frac{1}{Gk}(\ln G+\psi(2) & \frac{1}{k^2} A \end{bmatrix} \quad (21)$$

where

$$A = 1 + \psi'(2) + [\ln G + \psi(2)]^2 \quad (22)$$

The psi function which was previously mentioned and evaluated is also defined as

$$\psi(a) = \frac{d}{da} \ln \Gamma(a) \quad (23)$$

and its derivative is

$$\psi'(a) = \frac{d^2}{da^2} [\ln \Gamma(a)] \quad (24)$$

This information matrix has a determinant

$$\Delta = \frac{1}{G^2 k^2} [1 + \psi'(2)] \quad (25)$$

which is positive definite for all values of G and k. The log likelihood function for f(x; G, k) is

$$L(x) = n \ln k - n \ln G + (k-1) \Sigma \ln x - \frac{1}{G} \Sigma x^k \quad (26)$$

(Ref 23:52-54).

Sequential Tests of Hypotheses

In general, a test of hypotheses can provide three results: (1) the correct decision, (2) a Type One error which rejects the null hypothesis when it is true, [P(Type One Error) = α], and (3) a Type Two error which accepts the null hypothesis when it is false, [P(Type Two Error) = β]. It is possible to compute the probability of each type of error. It is also possible to determine the probability that a sequential test will terminate by stage n (Ref 8: 39-40). A statistical hypothesis is a statement about a distribution of the random variable(s). If the statistical hypothesis completely specifies the distribution, it is called simple. If the distribution is not completely specified it is called composite. An example of a simple hypothesis is $\theta=\theta_i$ where the parameter θ is the only parameter (Ref 14:268). The more common example of a composite hypothesis is $\theta>\theta_i$. This test allows a range rather than a point estimation and, therefore, does not completely specify the distribution. In this thesis, a different version of the composite hypothesis is used. It states $\theta=\theta_i$ where a second parameter (in this case, k) is unknown.

Sequential Probability Ratio Test

The Wald Sequential Probability Ratio Test (SPRT) provides the background for the tests to be conducted. The SPRT primarily involves tests of simple hypotheses. Random sample units are selected sequentially and, after each

additional observation, the test is conducted. Each test results in acceptance of the hypothesis, rejection of the hypothesis and acceptance of the alternate hypothesis, or no decision. If no decision is made, an additional observation is taken and the test is conducted again. The decision is made by determination of the test statistic:

$$Z_n = \prod_{i=1}^{n} f_1(x_i)/f_0(x_i) \tag{27}$$

where $f_0(x_i)$ is the probability density function assuming H_0: $\theta=\theta_0$ to be true and $f_1(x_i)$ is the probability density function assuming the alternate, H_1: $\theta=\theta_1$, to be true. The decision rules are:

1. If $Z_n \leq \beta/(1-\alpha)$, accept H_0.

2. If $Z_n \geq (1-\beta)/\alpha$, reject H_0 and accept H_1.

3. If $\beta/(1-\alpha) < Z_n < (1-\beta)/\alpha$, take another observation.

α and β have been previously defined under Sequential Tests of Hypotheses and are considered to be the pre-assigned risks. α is frequently termed the producer's risk because it is the probability that the alternate hypothesis is accepted when the null hypothesis is true. β is usually called the consumer's risk because it represents the probability of accepting the null hypothesis when it is false. It should be noted that the null hypothesis for the SPRT normally represents the preferable parameter value.

Figure 2 is a graphical representation of a typical untruncated SPRT where a decision to reject H_0 is made at the sixth observation.

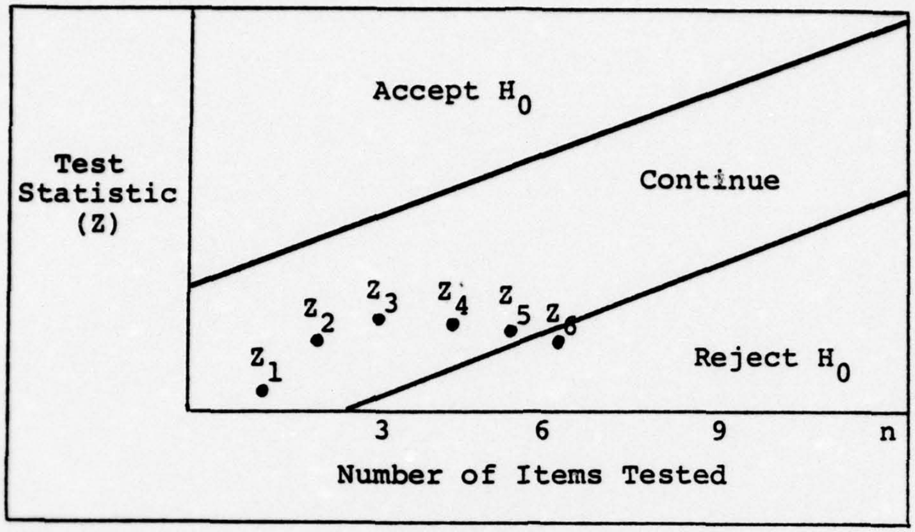

Fig. 2. SPRT

Tests

Three tests will be conducted. All have the same null and alternate hypotheses:

$$H_0\colon \theta=\theta_0 \tag{28}$$

$$H_1\colon \theta=\theta_1 \tag{29}$$

The values of the parameters θ, G, and k, will be unknown and will be estimated by their maximum likelihood estimators. An unknown parameter that is not part of the hypothesis (such as k) is called a nuisance parameter.

The tests will be modifications of the likelihood ratio test. The reason for conducting a likelihood ratio test is that the Weibull function cannot be transformed into any of the standard underlying distributions such as the Normal, Chi-square or F distributions which are extensively tabled test statistics. Adapting sequential procedures to the likelihood ratio test will provide an attempt to gain the

advantages described in Wald's SPRT.

Test One will be an application to the Weibull distribution of a sequential likelihood test devised by Cox (Ref 6:5-12). This test will use a test statistic based upon θ and k that is asymptotically equivalent to the likelihood ratio test statistic. The background and general procedures for Cox's test will be given in Chapter II. The acceptance and rejection boundaries are based upon Wald's SPRT but also include a coefficient based upon the information matrix. This coefficient will be derived and, for Test One, shown to be a constant larger than one. This coefficient expands the boundaries to account for the variance-covariance factors of the parameter estimations.

Test Two will be explained in Chapter IV. It will be similar to the first test by also being an asymptotically equivalent test. The test statistic differs by using the general scale parameter, G, with the shape parameter, k. The boundary coefficient will be derived and shown to be a variable. This means that the linear boundaries of the SPRT and Test One will be replaced by non-linear boundaries.

Test Three uses the exact sequential likelihood ratio test statistic for G and k instead of the asymptotically equivalent statistic. The variable boundaries from Test Two will be used with this test. Although Tests Two and Three use the general scale parameter in the statistic, the scale parameter, θ, is still the hypothesized test parameter. Test Three is explained in Chapter V.

Analysis

Assumptions. This thesis will be concerned only with Weibull distributions. It is assumed that the distribution to be tested is known to be Weibull. No method of testing for the type of distribution will be provided. The assumption setting the location parameter equal to zero has already been discussed. The scale parameter, θ, is assumed to be θ_0 or θ_1. Data was collected to show the results when this assumption is not true. This analysis will be given in Chapter VIII. The Monte Carlo method of obtaining failure times assumes that the simulation provides an adequate approximation of real events. No samples will be censored. Additional assumptions concerning numbers of sample runs and the initial estimation of the parameter, k, are embodied and specified within the methodology.

Standards. The only criterion to be used to judge the feasibility of the tests is analysis of the results. How do the actual alpha and beta errors compare with the desired alpha and beta errors? How many samples must be taken to achieve the desired results? Failure times are obtained by Monte Carlo simulation and tests were conducted over a varied range and combination of parameter values. Since the true input parameter values are known in the simulation, it becomes a matter of observing the percentage of correct decisions and the average sample numbers required to make the decisions. These observations are then compared

with the input alpha and beta errors and the calculated expected sample numbers.

II. Cox Sequential Likelihood Ratio Test

This section begins with a derivation of the Cox test statistic and boundary coefficient for continuous distributions. A previous validation using a non-Weibull distribution will be discussed.

Cox Asymptotic Test Statistic

The problem Cox was attempting to solve was to formulate a general sequential test of a parameter in the presence of a nuisance parameter. For ease of illustration, the general parameter to be tested and the nuisance parameter will be symbolized by the Weibull parameters of interest, θ and k, respectively. The value of k is unknown. Cox replaces Wald's statistic where k is known,

$$Z_n = L_n(x_n; \theta_1, k_0) - L_n(x_n; \theta_0, k_0), \tag{30}$$

where L_n is the log likelihood function, with a statistic using the maximum likelihood estimate of k, $\hat{k}$,

$$Z_n = L_n(x_n; \theta_1, \hat{k}) - L_n(x_n; \theta_0, \hat{k}). \tag{31}$$

This statistic is expanded about the true point, (θ,k):

$$\begin{aligned} Z_n = {} & (\theta_1-\theta_0) \frac{\partial L_n(x_n,\theta,k)}{\partial\theta} \\ & + \frac{1}{2}(\theta_1-\theta_0)(\theta_1+\theta_0-2\theta) \frac{\partial^2 L_n(x_n,\theta,k)}{\partial\theta^2} \\ & + (\theta_1-\theta_0)(\hat{k}-k) \frac{\partial^2 L_n(x_n,\theta,k)}{\partial\theta\,\partial k} \end{aligned} \tag{32}$$

Cox states that the statistic is asymptotically equivalent to Wald's statistic in Eq (30) if, and only if,

$$\frac{1}{n}\,\frac{\partial^2 L_n(x_n;\theta,k)}{\partial\theta\,\partial k} \to 0 \tag{33}$$

in probability. This convergence means that $\hat{\theta}$ and $\hat{k}$ are asymptotically independent (Ref 6:5-7). By Monte Carlo testing, Harter and Moore have shown that these parameters appear to be asymptotically independent based upon sample sizes as small as fifty with a fixed sample test (Ref 12:563). This thesis will look at much smaller minimum sample sizes than the fixed sample sizes used by Harter and Moore.

Using maximum likelihood theory on the terms of Eq (25),

$$-\frac{\partial^2 L_n(x_n,\theta,k)}{\partial\theta^2} \sim nI_{\theta\theta} \tag{34}$$

$$-\frac{\partial^2 L_n(x_n,\theta,k)}{\partial k^2} \sim nI_{kk} \tag{35}$$

$$-\frac{\partial^2 L_n(x_n,\theta,k)}{\partial\theta\,\partial k} \sim nI_{\theta k} \tag{36}$$

where

$$I_{\theta\theta} = E\left[-\frac{\partial^2 \log f(x_j;\theta,k)}{\partial\theta^2}\right] \tag{37}$$

$$I_{kk} = E\left[-\frac{\partial^2 \log f(x_j;\theta,k)}{\partial k^2}\right] \tag{38}$$

$$I_{\theta k} = E\left[-\frac{\partial^2 \log f(x_j;\theta,k)}{\partial\theta\,\partial k}\right] \tag{39}$$

It should be noted that the right sides of Eqs (37, (38), and (39) are the definitions of the respective entries in the general information matrix, Eq (10).

$\hat{\theta}$ and $\hat{k}$ satisfy asymptotically the equations

$$I_{\theta\theta}(\hat{\theta}-\theta) + I_{\theta k}(\hat{k}-k) = \frac{1}{n} \frac{\partial L_n(x_{n_i}, \theta, k)}{\partial \theta} \tag{40}$$

$$I_{\theta k}(\hat{\theta}-\theta) + I_{kk}(\hat{k}-k) = \frac{1}{n} \frac{\partial L_n(x_{n_i}, \theta, k)}{\partial k}. \tag{41}$$

Substituting these maximum likelihood estimates into Eq (31), the statistic to be used becomes

$$Z_n = (\theta_1-\theta_0)\ nI_{\hat{\theta}\hat{\theta}}\ [\hat{\theta} - \frac{1}{2}(\theta_0+\theta_1)]. \tag{42}$$

The expected value of $n\hat{\theta}$ is

$$E(n\hat{\theta}) = n\theta \tag{43}$$

and the variance is

$$V(n\hat{\theta}) = nI_{\hat{k}\hat{k}}/(I_{\hat{\theta}\hat{\theta}}I_{\hat{k}\hat{k}} - I^2_{.\hat{\theta}\hat{k}}). \tag{44}$$

The stochastic process (Z_n) is a random walk with mean increment per step of $\theta - \frac{1}{2}(\theta_1+\theta_2)$ and variance per step of C_n where

$$C_n = \left[1 - \frac{I^2_{\hat{\theta}\hat{k}}}{I_{\hat{\theta}\hat{\theta}}I_{\hat{k}\hat{k}}}\right]^{-1} \tag{45}$$

This allows use of Wald's theory for normally distributed observations (Ref 6:7).

<u>Cox Asymptotic Boundaries</u>

Boundaries are developed from Wald's original stopping boundaries. The Cox boundaries become (Ref 6:8)

$$C_n \log \left(\frac{\beta}{1-\alpha}\right), \quad C_n \log \left(\frac{1-\beta}{\alpha}\right) \qquad (46)$$

It can be seen from Eq (40) that increases in the variance coefficient, C_n, will widen the distance between the acceptance and rejection boundaries. It is likewise apparent that decreasing the discrimination ratio, θ_1/θ_0, will have a similar effect by decreasing the absolute value of the test statistic, Z_n.

A Validation of Cox's Test for the Normal Distribution

Cox attributes his test to an earlier method by Bartlett. The main difference is that Cox estimates the nuisance parameter solely on information from the random variable, x_i, while Bartlett makes two estimates of the nuisance parameter, $\hat{k}_0$ and $\hat{k}_1$, based upon the random variable, x_i, and the given value of each of the test parameters, θ_0 and θ_1 (Ref 4:239-244). Joanes compared the general methods of Bartlett and Cox with the more specific one-sided sequential T-tests of Wald and Barnard for the normal distribution. Interestingly, in his initial publication, Joanes failed to use the Cox coefficients on the stopping boundaries and declared that the Cox and Bartlett tests were not equivalent. After receiving correspondence from Cox, a correction was published three years later which showed equivalency between the two methods. Joanes was testing a normal distribution with H_0: $\mu=0$ and H_1: $\mu=\sigma$. The operating characteristic function and the average sample numbers of the Bartlett and Cox tests appeared

nearly as good as the results of the specialized Wald and Barnard tests. The primary difference between Cox/Bartlett and Wald/Bartlett was for true mean values near the middle of the range between the hypothesized values. In this region Cox/Bartlett tests were less powerful (Ref 15:633-637; 16:221). The Bartlett test was not pursued in this thesis because it is slightly more computationally involved than the Cox test. Furthermore, since it is an equivalent test, similar results would be expected.

III. Cox's Asymptotic Sequential Likelihood Test with the Weibull Distribution, $f(x;\theta,k)$

This section contains the mathematical formulation of the test, the methodology of the procedure to be used, and a discussion of its salient features.

Derivation of Test One

The general statistic to be used was given by Eq (42) in Chapter II. It is repeated here:

$$Z_n = (\theta_1 - \theta_0)\ nI_{\hat{\theta}\hat{\theta}}\ [\hat{\theta} - \tfrac{1}{2}(\theta_0 + \theta_1)] \qquad (42)$$

It can be observed that the general test statistic can be used directly for the Weibull distribution. The value for $\hat{\theta}$ is obtained from Eq (9) after solving for $\hat{k}$. $I_{\hat{\theta}\hat{\theta}}$ will be defined in Eq (50). The two constants, θ_0 and θ_1, will be the hypothesized input values. It should be noted that contrary to the more common practice, the alternate is larger than the null parameter value. This also reverses the alpha and beta errors from their standard form but is consistent with the general Cox test procedure. The boundaries are computed from the general form, accept if

$$Z_n < C_n \log\left(\frac{1-\beta}{\alpha}\right) \qquad (47)$$

and reject if

$$Z_n > C_n \log\left(\frac{\beta}{1-\alpha}\right) \qquad (48)$$

From Eq (45) and using maximum likelihood estimates,

$$C_n = \left[1 - \frac{I^2_{\hat{\theta}\hat{k}}}{I_{\hat{\theta}\hat{\theta}} I_{\hat{k}\hat{k}}} \right]^{-1} \tag{49}$$

with $I_{\hat{\theta}\hat{\theta}}$, $I_{\hat{k}\hat{k}}$, and $I_{\hat{\theta}\hat{k}}$ defined generally in Eqs (37), (38), and (39). These equations and the likelihood information matrices, Eqs (10) and (11) provide the solution for C_n:

$$I_{\hat{\theta}\hat{\theta}} = k^2 \tag{50}$$

$$I_{\hat{\theta}\hat{k}} = \Gamma'(2.0) \tag{51}$$

$$I_{\hat{k}\hat{k}} = \frac{1}{k^2} + \frac{\Gamma'(2.0)}{k^2} \tag{52}$$

$$C_n = \left[1 - \frac{[\Gamma'(2.0)]^2}{k^2 [\frac{1}{k^2} + \frac{\Gamma''(2.0)}{k^2}]} \right]^{-1} \tag{53}$$

Substituting the values of $\Gamma'(2.0)$ and $\Gamma''(2.0)$ obtained from Eqs (14), (15), and (16):

$$C_n = \left[1 - \frac{(.6449340668)^2}{1.8236806608} \right]^{-1} = 1.295466348 \tag{54}$$

Methodology

The steps to be used for a single test are:

1. Generate a given minimum number of failure times using the null value of the scale parameter and a given value of the shape parameter for the two parameter Weibull function.

2. Estimate the shape parameter by solving Eq (7) using the Newton-Raphson iteration method.

3. Estimate the scale parameter directly with Eq (9) using the maximum likelihood estimate of the shape parameter.

4. Calculate the Cox test statistic using the estimate obtained in Steps 2 and 3.

5. Multiply the constant, C_n, times the standard Wald boundaries.

6. Perform the comparison of the test statistic with the boundaries. If a boundary is exceeded, the appropriate hypothesis is chosen, the number of samples is noted, and the single test is complete. If neither boundary is exceeded, continue with Step 7.

7. Generate one additional sample failure time.

8. Re-estimate the shape and scale parameters as in Steps 2 and 3 using the original plus the added failure times.

9. Re-accomplish Steps 4-6 and continue with the loop of steps until a decision is made or a truncation point of four hundred is reached.

10. If the truncation point is reached, select the hypothesis whose calculated boundary is closer to the value of the test statistic.

The single test covered above is repeated five hundred times for each combination of θ_0, θ_1, and k at each input

risk level. The percentage of times the alternate hypothesis is chosen becomes the alpha error for the specific combination of parameter values. The total number of failure samples generated by all tests resulting in acceptance of the null hypothesis is divided by the number of acceptances to obtain the average sample number to accept under H_0. Similarly the average sample number to reject under H_0 is computed. The number of tests which were truncated are recorded for acceptance or rejection.

The steps and procedures listed above are then repeated with sample failure times generated with the alternate scale parameter in Step 1. The beta error is the percentage of tests that accept H_0 under the alternate hypothesis. The average sample numbers to accept and reject and the number of tests resulting in truncation are recorded by the same procedure used under the null hypothesis.

The choice of four hundred as a truncation point was felt to be sufficiently larger than the expected sample number in all test cases. It would, therefore, approximate an untruncated test.

Generation of Failure Times

The required number of random variates, $F(x_i)$, uniformly distributed from zero to one, were generated as needed with the IMSL routine GGUBF (Ref 15). The random number generator was seeded for each run with a two digit random number.

These random variates were changed into Weibull failure times, x_i, by the transformation

$$x_i = \theta \cdot [-\ln(1-F(x_i))]^{1/k} \tag{55}$$

where $F(x_i)$ denotes the cumulative distribution function of the uniform distribution. The entire Monte Carlo simulation was conducted on the CDC 6600 computer system.

Newton-Raphson Procedure

Given the function, $f(k)$, for the maximum likelihood of k in Eq (7) and its derivative, $f'(k)$, set

$$f(\hat{k}) = \hat{k} - f(\hat{k})/f'(\hat{k}) \tag{56}$$

Let p_0 be an approximation to a solution of $f(k)=0$. Generate the sequence $[p_n]$ recursively by the relation $p_n=g(p_{n-1})$, n=1,2.... Continue solving for p_n until (p_n-p_{n-1}) is less than a specified tolerance (Ref 20:19-24). The tolerance used for the computer program is .0000005. Eq (56), after appropriate substitution becomes

$$g(k) = k + \frac{[n/\hat{k}+\Sigma \ln x-(1/n\ \Sigma(x^{\hat{k}} \ln x))]}{[n\hat{k}^{-2}+(\hat{k}/n)(\Sigma x^{\hat{k}} \ln x)+\Sigma x^{\hat{k}}\cdot\Sigma x^{\hat{k}-1} \ln x]} \tag{57}$$

An initial estimate of $\hat{k}$ must be provided to the Newton-Raphson algorithm. The input value of k was used for this estimate. This value would, of course, be unknown in an actual test. It is used in the thesis to decrease computer time by providing quicker convergence. Small tests were conducted with arbitrary estimates of k other than the actual

value. These tests all converged reasonably quickly. An upper limit of three hundred was placed upon the number of convergence iterations but the limit was never reached. As each additional observation is taken for the sequential procedure, the value of $\hat{k}$ from the preceding sequential test is used as the initial estimate for the next test. The number of iterations on the first sequential test was about ten. The subsequent tests using the previous estimate usually converged within two iterations.

IV. Cox's Asymptotic Sequential Likelihood Test with the Weibull Distribution, $f(x;G,k)$

This chapter will explain only the differences in the derivation and methodology from that given in Chapter III.

Derivation of Test Two

The log likelihood function used as a basis for this test is given in Eq (26). The general test statistic, Eq (42), will be used with parameters G and k. Given Eq (9) for the maximum likelihood of θ and the relationship $\hat{G} = \hat{\theta}^{\hat{k}}$,

$$\hat{G} = \frac{1}{n} \Sigma \, x^{\hat{k}}. \tag{58}$$

Solving Eq (45) for C_n by use of the entries in the information matrix, Eq (21) and Eqs (37), (38) and (39):

$$I_{\hat{G}\hat{G}} = \frac{1}{\hat{G}^2} \tag{59}$$

$$I_{\hat{G}\hat{k}} = -\frac{1}{\hat{G}\hat{k}} [\ln \hat{G} + \psi(2)] \tag{60}$$

$$I_{\hat{k}\hat{k}} = \frac{1}{\hat{k}^2} [1 + \psi'(2) + (\ln \hat{G} + \psi(2))^2]. \tag{61}$$

Now using the digamma values given in Eqs (17) and (18):

$$C_n = \left[1 - \frac{\frac{1}{\hat{G}^2\hat{k}^2} [\ln(\hat{G}) + .422784351]^2}{\left(\frac{1}{\hat{G}^2}\right)\left(\frac{1}{\hat{k}^2}\right) [1.6449340668 + (\ln(\hat{G}) + .422784351)^2]} \right]^{-1}$$

which reduces to

$$C_n = \left[1 - \frac{\ln^2\hat{G}+.84556867 \ln \hat{G}+.17846594}{\ln^2\hat{G}+.84556867 \ln\hat{G}+1.8236806608} \right]^{-1} \quad (62)$$

The variable, C_n, must be recomputed each time that a new $\hat{G}$ and $\hat{k}$ are estimated. Using the value of $I_{\hat{G}\hat{G}}$ obtained in Eq (59) and letting $G_i = \theta_i{}^k$, the test statistic becomes

$$Z_n = (\theta_1{}^{\hat{k}} - \theta_0{}^{\hat{k}})\left(\frac{1}{\hat{\theta}^{\hat{k}}}\right)^2 n \left(\hat{\theta}^{\hat{k}} - \frac{1}{2}(\theta_0{}^{\hat{k}} + \theta_1{}^{\hat{k}})\right) \quad (63)$$

Methodology

The specific changes to the steps listed in Test One are given below:

3. Estimate the scale parameter, $\hat{G}$, directly from Eq (58).

5. Calculate the value for C_n from Eq (62). Multiply C_n times the standard Wald boundaries.

V. An Exact Likelihood Ratio Test with Cox Boundaries

This chapter will derive the test statistic and list differences from Test One.

Derivation of Test Three

The statistic to be used in the sequential likelihood ratio for the G parameter with k replaced by $\hat{k}$. Starting with the f(x;G,k) version of the log likelihood equation, Eq (25),

$$
\begin{aligned}
Z &= \mathrm{Ln}(x;G_1,\hat{k})/\mathrm{Ln}(x;G_0,\hat{k}) \\
&= [n \ln \hat{k} - n \ln G_1 + (\hat{k}-1)\Sigma \ln x_i - \frac{1}{G_1} \Sigma x^{\hat{k}}] \\
&\quad - [n \ln \hat{k} - n \ln G_0 + (\hat{k}-1)\Sigma \ln x_i - \frac{1}{G_0} \Sigma x^{\hat{k}}] \\
&= n \ln(G_0/G_1) - (\frac{1}{G_0} - \frac{1}{G_1}) \Sigma x^{\hat{k}} \qquad (64)
\end{aligned}
$$

This statistic will be referred to as the "exact" test statistic. It will use the variance coefficient, C_n, from Chapter IV.

Methodology

The specific changes to the steps listed in Chapter III are given below:

3. Estimate the scale parameter, $\hat{G}$, directly from Eq (58).

4. Calculate the exact test statistic from Eq (64) using the scale parameter estimate obtained in Step 3.

5. Calculate the value for C_n from Eq (62). Multiply C_n times the standard Wald boundaries.

VI. Input Values and Results

Input Values

Input alpha and beta risks were chosen to provide tests similar to MIL-STD-781B for exponential SPRT's. For the untruncated tests, the values of the desired risks used were .2, .15, .1, and .05.

The values of θ were $\theta_0=1$ and $\theta_1=1.5$ and 2. These choices result in the commonly used discrimination ratios, θ_1/θ_0. The SPRT with known k allows substitution of other values of θ because of the relationship, $\theta_1^{k_1} = \theta_2^{k_2}$ (Ref 28:34-35). This relationship does not hold for Test One but is applicable to Tests Two and Three. In Test One, θ and k appear in forms other than θ_k which is the basis for the substitution. In Tests Two and Three, the parameters are always of the form, θ^k, in both Z_n and C_n. Normalized time units are assumed for θ values.

The values of the shape parameter, k, are .5, .75, 1.0, 1.5, 2.0, and 3.0. These, also, are representative of frequently occurring values in actual Weibull distributions.

The minimum sample size for the tests was varied between 3 and 20. Since the thesis is essential an initial look at these methods, the minimum sample size was cut off whenever smaller sizes appeared to be unproductive because of large output risks. Increases in the minimum sample size were terminated when output risks were close to the

desired risks. The tabulated data will specify the minimum sample size used for each test run. Because of the asymptotic nature of the tests, minimum sample sizes larger than those needed for an SPRT would be expected. Table I lists the minimum sample values used for each test.

Table I

Minimum Sample Sizes

Test	θ_1	Minimum Sample Size
1	1.5	5
	2.0	3,5
	2.0	5*
2	1.5	5,10
	2.0	5,10
3	1.5	5
	2.0	3,5

* Test run using input beta = 2•alpha(β=2α)

Results

The output tables for the untruncated tests are listed in Appendix B. Most of the analysis can be accomplished directly by reading the tables. This paragraph will note general findings. The expected sample number, E(n), in the tables is based on θ_0 and provides a basis of comparison for the average sample numbers.

Error. The Monte Carlo method assumes perfect random numbers and a perfect system model. This still allows errors due to the basic laws of statistics. These statistical

errors are portrayed by the output alpha and beta errors. These errors are expected to decrease proportionately with $1/\sqrt{n}$ where n is the number of Monte Carlo repetitions (Ref 24:259). In the case of sequential trials, this error relationship is more approximate because the number of trials differs in each run. The use of five hundred repetitions did not completely dampen out all the abberations in the alpha and beta errors. Examination of the output tables in Appendix B will show occasions where the output error for a smaller input error is larger than the output error for a larger input error. This is contrary to expected results because the smaller input risks have wider boundaries. An example of this can be seen in Test 3, Table III-1, θ_1=1.5,k=.5 and a minimum sample size of 3. In this case the output error for an input of .15 is .454 with an average sample number of approximately 25. The output beta error for an input beta of .1 is .504 with an average sample number of approximately 40. Most of these variations seen in the lower range of the k values are due to small Monte Carlo size. Central processing time, particularly for low k values, was rather large. For example with θ_1=1.5, k=.5 and input risks of .05, the central processing time for some of the tests exceeded 3000 seconds. It was, therefore, not deemed worthwhile to extend the trials to obtain more consistent data. Another source of error is error within the computer. An example would be round-off error. For the degree of accuracy needed for this

thesis, round-off error was not considered. Error due to the estimation procedure will be discussed in the next subparagraph.

Estimation of Parameters. Since all three tests used the same maximum likelihood estimation procedures, the estimation results will be discussed jointly for the three tests. Appendix E provides sample values of the means of the estimated parameters and their associated average sample numbers. Estimates of k were consistently higher than the real values. The estimates were generally closer for $\theta_1=1.5$ than for $\theta_1=2.0$. This does not appear to be strictly a function of the average sample number. This can be seen by comparing estimates for different risk levels but the same θ and k. In these cases, a rise in sample size with decreasing risk level does not obviously correspond to improved estimates. Estimates of k were slightly higher for an input $\theta=\theta_1$ as opposed to $\theta=\theta_0$. The θ estimates were much closer than the k estimates. The G estimates differ from their true values because of the bias of the k estimator.

A comparison of the expected sample number, E(n), with the average sample numbers in Appendix B shows the average sample numbers to be lower for Test One and higher than the calculated E(n) for Tests Two and Three. Calculation procedures for E(n) will be set forth in Chapter VII. E(n) for these untruncated tests was based upon the actual

value of k. The calculated E(n) does not anticipate the use of minimum sample sizes but rather allows the test to terminate after every observation. Minimum sample sizes are used to increase the accuracy of the test by dampening the affects of outlying data points early in the test. Use of minimum sample sizes will tend to increase the average sample numbers.

Test One. This test had the lowest average sample numbers of the three tests. For values of E(n) larger than the minimum sample size, the average sample numbers were less than E(n). In the lower values of k, the sum of the output risks was the highest. A bias to accept H_0 can be seen from the data. Table B-I-4 in Appendix B shows an attempt to balance the output risks. The input beta error was doubled to increase the output alpha error. Instead of a reduction in the alpha error, a decrease in the average sample number occurred. Output alpha error showed little change.

Test Two. This test was strongly biased to accept H_0 for all k values. The average sample number to Reject H_0 when H_1 was true were higher than the other tests.

Test Three. The least bias to accept or reject was seen in this test. Particularly good results were noted in the output risks for low k values, $\theta=2$ and a minimum sample size of 5. The output risks compared favorably with the input risks.

VII. Truncated Tests

The method of determination of the expected sample number, the truncation procedure, and the analysis of test results will be given.

Expected Sample Numbers

The expected sample number is derived from the operational characteristic function:

$$P(\theta) \approx \frac{A^{h(\theta)} - 1}{A^{h(\theta)} - B^{h(\theta)}} \tag{65}$$

where

$$A = (1-\beta)/\alpha \tag{66}$$

$$B = \beta/(1-\alpha) \tag{67}$$

and

$h(\theta)$ is the solution of

$$\int_{-\infty}^{\infty} \left[\frac{f(x,\theta_1)}{f(x,\theta_0)}\right]^{h(\theta)} f(x,\theta) = 1 \tag{68}$$

$$P(\theta_0) = 1-\alpha \tag{69}$$

$$P(\theta_1) = \beta \tag{70}$$

Eq (65) can now be solved for $h(\theta_0)$ and $h(\theta_1)$ given $P(\theta_0)$ and $P(\theta_1)$. Let

$$d = (1/\theta_0{}^k) - (1/\theta_1{}^k) \quad (71)$$

$$s = \frac{k \ln(\theta_0/\theta_1)}{d} \quad (72)$$

$$h_0 = \frac{\ln[(1-\alpha)/\beta]}{d} \quad (73)$$

$$h_1 = (\ln A)/d \quad (74)$$

Now that $P(\theta)$ can be solved, the expected value of n, $E(n)$, is

$$E(n|\theta = [P(\theta)\ln B + (1-P(\theta))\ln A]/E(Z|\theta) \quad (75)$$

where $E(Z|\theta) = \int_0^{\infty} \ln \frac{f(x,\theta_1)}{f(x,\theta_0)} f(x,\theta)\, dx$

$$= \ln(\theta_0/\theta_1) + (\theta/\theta_0) - (\theta/\theta_1) \quad (76)$$

Eq (75) becomes:

$$E(n|\theta) = \frac{h_1 - (h_0+h_1)\ P(\theta)}{S - \theta^k} \quad (77)$$

Using Eqs (69) and (70)

$$E(n|\theta_0) = \frac{\alpha(h_0+h_1) - h_0}{S - \theta_0{}^k} \quad (78)$$

and

$$E(n|\theta_1) = \frac{h_1 - \beta(h_0+h_1)}{S - \theta_1{}^k} \quad (79)$$

Eqs (78) and (79) are the expected sample numbers for the SPRT. For the Weibull distribution with $\theta_1 > \theta_0$, $E(n|\theta_0)$ provides a larger value than $E(n|\theta_1)$. Since the actual tests have a nuisance parameter and are, therefore, less certain

than the SPRT, the larger value, $E(n|\theta_0)$, was used to provide a slightly larger number of observations before truncation.

Truncation Procedure

The method of truncation will be to terminate the test at a multiple of the expected sample number. This thesis will use a multiple of two. This method provides a straightforward vertical truncation that is unbiased by the truncation. Figure 3 shows Test One truncated at $2 \cdot E(n)$. Tests Two and Three would be the same except the accept and reject lines would not have a constant horizontal slope.

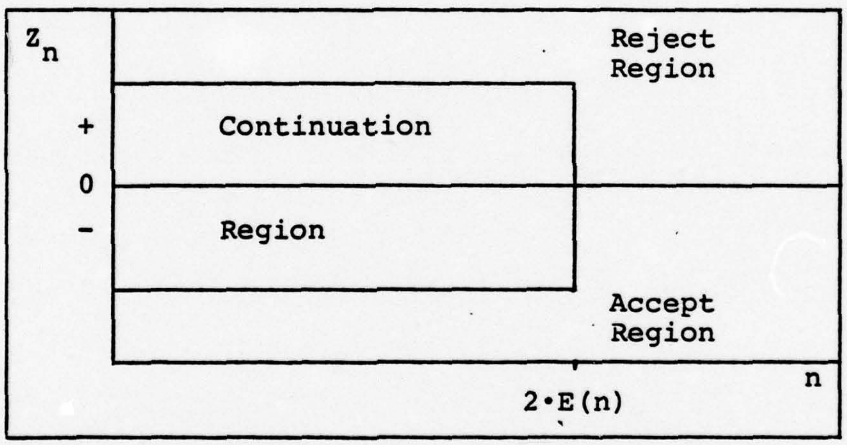

Fig. 3 Truncated Sequential Likelihood Ratio Test

The truncated test can be considered a compromise between a sequential test and a fixed sample test. Since k is unknown, it is not possible to calculate the truncation point prior to testing because E(n) is a function of k. This could be a disadvantage in the case of testing that is expensive or requires extensive scheduling. For the

Monte Carlo techniques, E(n) will be recomputed with each re-estimation of k or in other words, at each observation. The mean of E(n) for each of the five hundred trials will be listed for each run. Separate mean values will be listed for tests conducted under H_0 and H_1, $E(n)_0$ and $E(n)_1$, respectively. The minimum sample size was five for all tests. Input θ and k values were the same as the untruncated tests.

Results

Appendix C gives the results for Tests One, Two, and Three. With respect to the untruncated tests of Appendix B, results were as expected. The average sample numbers were less and the alpha and beta errors greater. For the larger k values, the average sample numbers will be greater than 2•E(n) because the minimum sample size of five was greater than the truncation point. These tests were actually fixed sample tests.

Table II lists the other changes that were noted by truncating.

Table II

<u>Comparison of Truncated and Untruncated Tests</u>

Test	θ_{IN}	Remarks
I	1.5	No change.
	2.0	Improved output risks.
II	1.5	No change.
	2.0	Less bias to accept H_0, improved β risk.
III	1.5	No change.
	2.0	Caused a bias to reject.

VIII. Power

The power of the test is the probability that θ_0 will be rejected when θ is the true parameter value. When $\theta=\theta_0$, the power equals the alpha error. When $\theta=\theta_1$, the power equals $1-\beta$. For the SPRT, Wald has devised a computational method of determining power. Eq (65) is the power equation. A more complete description can be found in almost all discussions of the SPRT. A recommended description is Mood and Graybills' (Ref 19:388-391). Because of the differences of the sequential likelihood ratio test, the computational method was not used. Instead, a Monte Carlo test was conducted with fourteen input values ranging from .1 to 2.6 for the discrimination ratios of 1.5 and 2.0. The same six values of k that were used in the prior tests were repeated. The tests were truncated at $2 \cdot E(n)$. The number of trial runs for each combination of inputs was two hundred. Input risks were limited to .1 and .2. The power was determined by subtracting the output beta error from one. Figures 4 and 5 provide graphical representations of the power of the three tests for the specified sets of inputs. These figures should not be construed to represent the power relationships between the tests for all inputs. The tabulated power for all the tests appears in Appendix D. Generally, all three tests showed typical power curves except that the errors at θ_0 and θ_1 were greater than designed values at low k values.

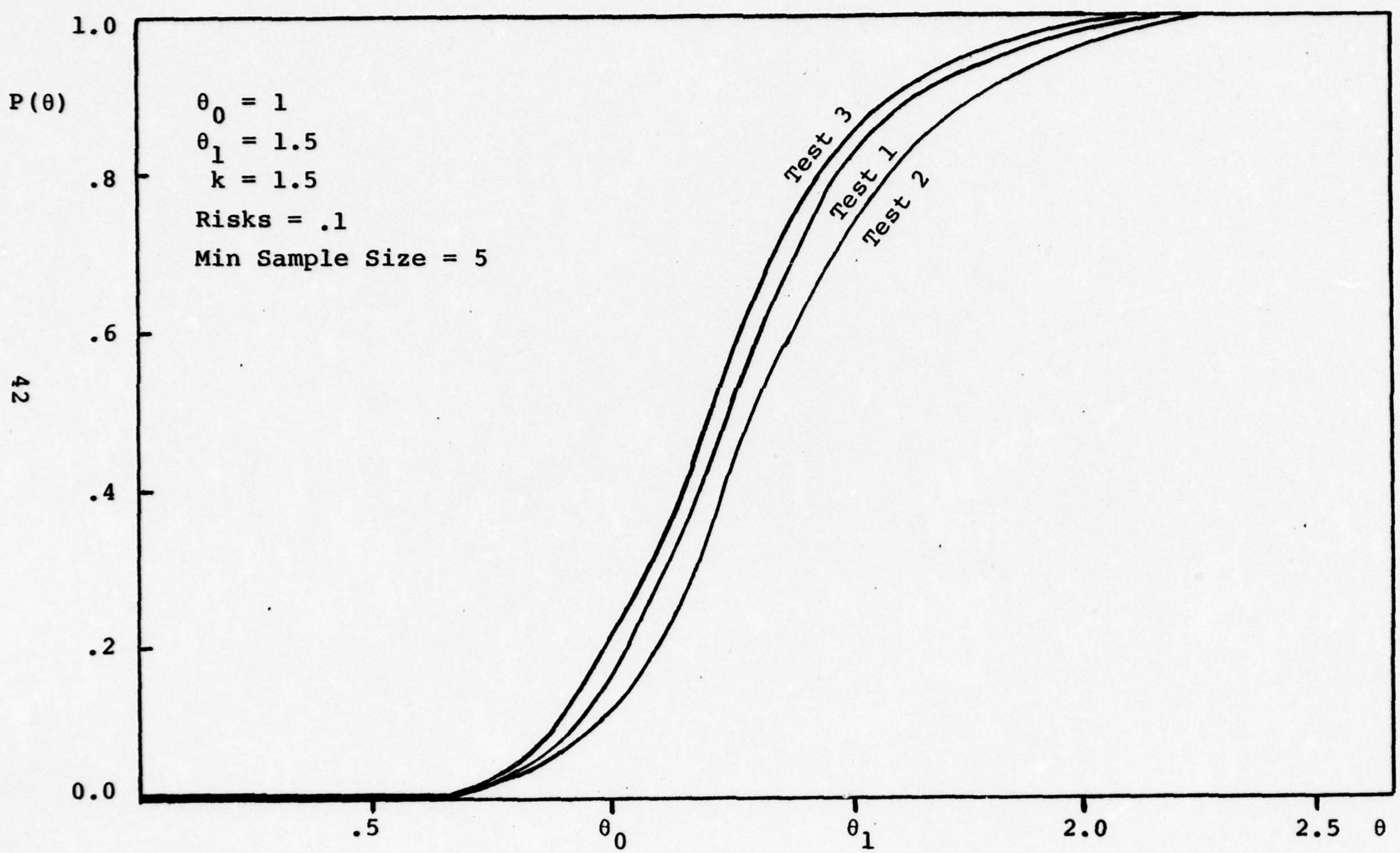

Fig. 4 Power, $\theta_1/\theta_0 = 1.5$

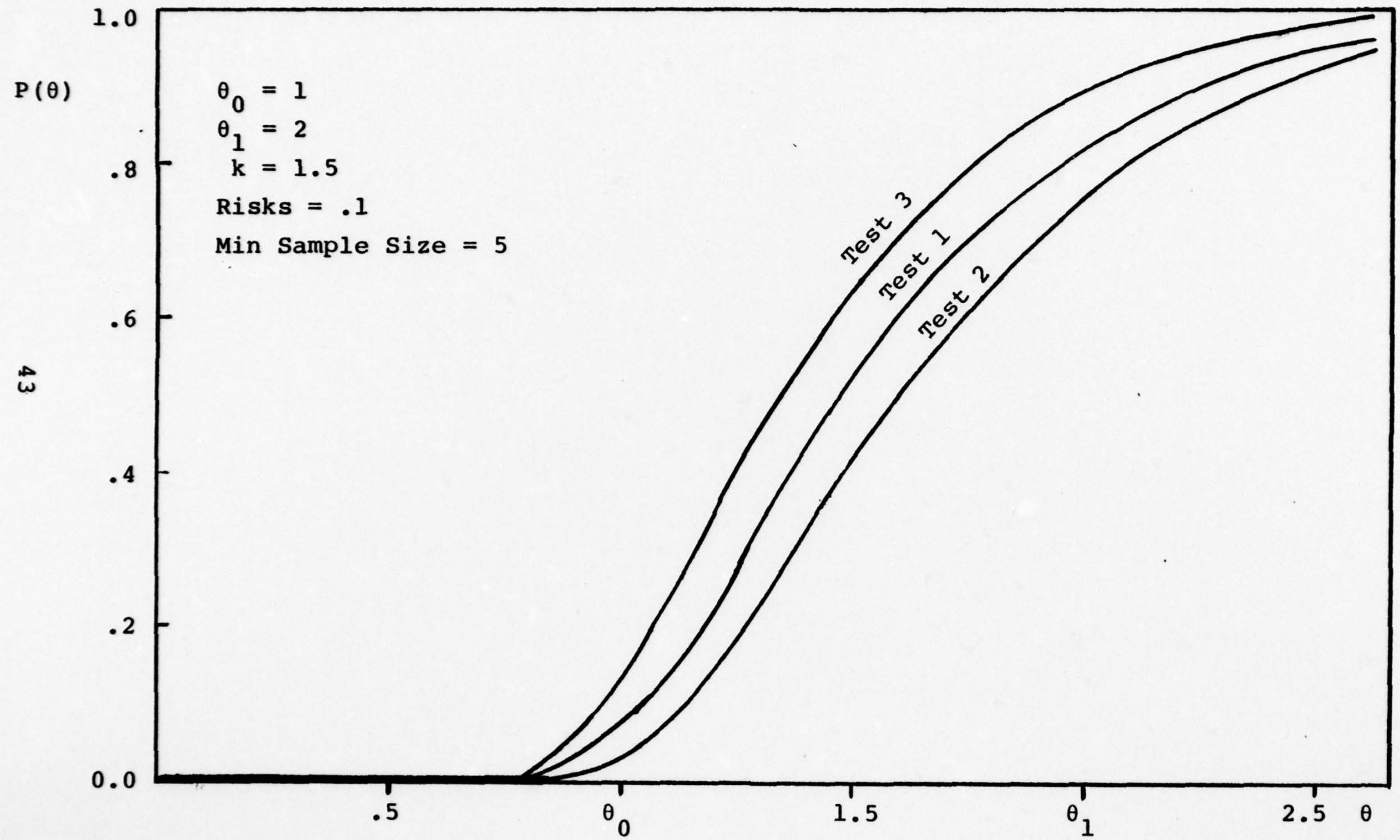

Fig. 5 Power, $\theta_1/\theta_0 = 2$

IX. Conclusions and Recommendations

Conclusions

It cannot be summarily stated that any one of the three tests is superior to the others in all facets. It can be said that all three tests showed similar results to those of the SPRT. Test One was notable for having the smallest average sample numbers. Test Three showed the least bias in the output risks. These two tests are probably the best candidates for further development. These basic tests provide a basis for trying adaptive procedures to improve the results. Any of the three tests could be used in their present form for the higher k values.

Recommendations

· Adaptive procedures are quite extensive in sequential testing and could provide significance improvements to the tests in this thesis. One adaptive procedure by Harter and Moore that is similar to Test Three has been published (Ref 11:100-104). This test used Wald boundaries with $\hat{k}$ corrected for bias and had smaller average sample numbers than Test Three. Other adaptive procedures that could be tried include sequentially decreasing the boundaries, decreasing the boundaries if the k estimate are greater than 1.0 or 1.5, experimenting with different combinations of input risks, and developing other functions of θ to

use as the asymptotic test statistic. Ghosh states that the given approximation for the boundary coefficient, C_n, is essentially a first approximation and that more refined approximations may be possible (Ref 8:333). The two most prominent objectives would be to improve the output risks for the low k values and to increase the output errors up to approximately the input errors for the higher k values in order to lower the average sample numbers. The first of these objectives is obviously the more difficult. Improving the k estimation procedure might provide improvement. Another estimation procedure which could be tried has been published by Wingo (Ref 29; Ref 30). This previously mentioned unbiasing factor of k could also be tried with these tests. It may be possible to combine tests, using one test statistic and boundary to accept and another to reject. These recommendations do not exhaust the list of possible adaptive procedures. They are offered only in the hope of inspiring further invesigation.

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APPENDIX A

COMPUTER PROGRAM

```
C SEQUENTIAL LIKELIHOOD RATIO TESTS WITH OPTION TO TRUNCATE
C PROGRAM REQUIRES ATTACHMENT OF IMSL LIBRARY TO OBTAIN RANDOM
C GENERATING FUNCTION GGUBF
C DISTINGUISHES BETWEEN WEIBULL DISTRIBUTIONS WITH RESPECT TO
C SCALE PARAMETER.  UNKNOWN SHAPE PARAMETER REPLACED BY ITS
C MAXIMUM LIKELIHOOD ESTIMATE.  MONTE CARLO SIMULATION
C TEST 1-ASYMPTOTIC SLRT-COX(1953) METHOD WITH THETA STATISTIC
C TEST 2-ASYMPTOTIC SLRT USING COX METHOD  WITH G STATISTIC
C TEST 3-EXACT SLRT WITH G STATISTIC USING COX BORY COEFFICIENT
C         ** DEFINITIONS **
C ISEED-SEED FOR RANDOM NUMBER GENERATOR
C IA - PARAMETER TO CHANGE INPUT ALPHA AND BETA ERRORS
C IV - PARAMETER CHANGING INPUT THETA TO ALTERNATE HYPOTHESIS
C IP-PARAMETER TO CHANGE SHAPE
C ASN - AVERAGE SAMPLE NUMBER
C (A) - ACCEPT
C (R) - REJECT
C ALPHA(IA)/BETA-INPUT RISKS
C RALPHA/RBETA - COMPUTED OUTPUT ERROR (ALPHA/BETA)
C NTRUNC - TRUNCATION POINT FOR SEQUENTIAL OBSERVATIONS
C MULT-TRUNCATION MULTIPLE OF E(N)
C NROBSA/NROBSR- NUMBER OF OBS UNTIL ACCEPT/REJECT
C NRACPT/NRRJCT-NUMBER OF TESTS RESULTING IN ACCEPT/REJECT
C NTRUNAC/NTRUNRJ-NR OF TESTS RESULTING IN TRUNCATION
C IQ - CUMULATIVE OBS TOTAL FOR EACH RUN
C TOL- TOLERANCE FOR NEWTON RAPHSON ITERATION
C SMALL/RLARGE-TOLERANCES TO ELIMINATE DATA THAT WILL ABORT RUN
C NUMOBO NR OF BAD OBS EXCEEDING SMALL/RLARGE LIMITS
C IQ1 - MINIMUM NR OF OBS FOR EACH RUN
C NSAMP-NR OF RUNS PER TEST
C                           **
      PROGRAM HOFF1(INPUT,OUTPUT)
      DIMENSION X(400),FX(400),NTRUNAC(4,2),NTRUNRJ(4,2)
      DIMENSION ASNA(4,2),ASNR(4,2),NUMOBO(4,2)
      DIMENSION NROBSA(4,2),NROBSR(4,2)
      DIMENSION RNROBSA(4,2),RNROBSR(4,2)
      DIMENSION RALPHA(4),RBETA(4)
      DIMENSION ALPHA(4),RKI(6)
      DIMENSION RKK(500),EETHETA(500),EKMEAN(4,2),THETAM(4,2)
      DIMENSION RNTRNC(500)
      DIMENSION VARK(4,2),VART(4,2),VARN(4,2),RNTRMN(4,2)
      DATA ALPHA(1)/.20/,ALPHA(2)/.15/,ALPHA(3)/.10/,
      DATA NSAMP/500/,THETA0/1.0/,NTRUNC/400/,ALPHA(4)/.05/
      DATA TOL/.0000005/,SMALL/1.0E-050/,RLARGE/2000./
      DATA RKI(1)/.5/,RKI(2)/.75/,RKI(3)/1.0/,RKI(4)/1.5/
      DATA RKI(5)/2.0/,RKI(6)/3.0/
      DATA THETA1/1.5/,IQ1/05/,ISEED/19/
```

```
      DATA IA1/1/,IA2/4/,IA3/1/,IP1/1/,IP2/5/,IP3/1/
      DATA MULT/02/
C       DO LOOP FOR CHANGING SHAPE PARAMETER (K)
      DO 111 IP=IP1,IP2,IP3
      RK0=RKI(IP)
      RK0INV=1./RK0
C LOOP FOR CHANGING ALPHA/BETA LEVEL
      DO 1 IA=IA1,IA2,IA3
      BETA=ALPHA(IA)
      A=ALOG((1.-BETA)/ALPHA(IA))
      B=ALOG(BETA/(1.-ALPHA(IA)))
C  CALCULATE NUMERATOR OF E(N) IF TRUNCATING ON MULTIPLE  OF E(N)
      RNUME=ALPHA(IA)*(ALOG((1-ALPHA(IA))/BETA)+A)-ALOG(1-ALPHA
     C(IA))/BETA
C LOOP FOR CHANGING INPUT THETA TO ALTERNATE
      DO 2 IV=1,2
      IF(IV.EQ.1)THETA=THETA0
      IF(IV.EQ.2)THETA=THETA1
      NTRUNAC(IA,IV)=0
      NTRUNRJ(IA,IV)=0
      NRACPT=0
      NRRJCT=0
      NUMOBO(IA,IV)=0
      NROBSA(IA,IV)=0
      NROBSR(IA,IV)=0
C MAKE NSAMP RUNS  .
      DO 3 IM=1,NSAMP
  23  IQ=IQ1
C LOOP FOR GENERATING INITIAL IQ1 RANDOM VARIATES
      DO 4 ID=1,IQ1
      FX(ID)=GGUBF(ISEED)
  04  CONTINUE
C LOOP TOCOMPUTE X-VALUES (FAILURE TIMES) FROM FX VALUES
      DO 5 IE=1,IQ1
      X(IE)=THETA*((-ALOG(1.-FX(IE)))**RK0INV)
  05  CONTINUE
C IF LARGEST X=0, THROW OUT SAMPLE AND TAKE A NEW ONE
      SUMM=0.00000
      DO 6 IZ=1,IQ
      SUMM=X(IZ)+SUMM
  06  CONTINUE
      IF(SUMM.GT.0.)GO TO 40
      NUMOBO(IA,IV)=NUMOBO(IA,IV)+1
      GO TO 99
C  ITERATIVE NEWTON-RAPHSON METHOD TO CALCULATE
C              MAX LIKELIHOOD ESTIMATE OF K1
C INITIAL VALUE OF K1 IS K0
  40  RK1=RK0
C LOOP FOR PERFORMING ITERATIONS
  41  DO 7 IF=1,300
      SUM1=0.
      SUM2=0.
```

```
      SUM3=0.
      SUM4=0.
C LOOP FOR CALCULATING SUMMATIONS
      DO 8 IG=1,IQ
C SUM OF X(I)**K1
      T1=X(IG)**RK1
      SUM1=SUM1+T1
C SUM OF LN(X(I))
      T2=ALOG(X(IG))
      SUM2=SUM2+T2
C SUM OF (X(I)**K1)*LN(X(I))
C SUM OF (X(I)**K1)*LN(X(I))
      SUM3=SUM3+T1*T2
C SUM OF (X(I)**K1)*((LN(X(I)))**2
      SUM4=SUM4+T1*(T2**2.)
  08  CONTINUE
C CALCULATE NUMERATOR OF F(K1) AND PART OF DENOMINATOR
      W1=IQ*SUM1
      W2=IQ*SUM3
C CALCULATE DENOMINATOR OF F(K1)
      W3=W2-SUM1*SUM2
C IF DENOMINATOR = 0, THROW OUT SAMPLE ,TAKE NEW ONE,CONTINUE
      IF (ABS(W3).LT.SMALL)GO TO 251
      W4=W3*W3
C CALCULATE FIRST DERIVATIVE OF F(K1)
      FPK=1.-((W3*W2-(W1*(IQ*SUM4-SUM3*SUM2)))/(W4))
C IF  FPK = 0 , DO SAME AS WHEN W3=0.
      IF(ABS(FPK).LT.SMALL)GO TO 351
C CALCULATE F(K1)
      FK=RK1-(W1/W3)
C CALCULATE GK FOR NEWTON RAPHSON METHOD
      GK=RK1-(FK/FPK)
C IF NEW VALUE OF K1 IS TOO LARGE, THROW OUT SAMPLE
      IF(GK.GT.RLARGE)GO TO 461
C IS NEW VALUE LESS THAN TOL ? IF YES,DEPART LOOP, OTHERWISE
C CONTINUE WITH NEW K1 = GK
      TEST=ABS(GK-RK1)
      IF(TEST.LE.TOL)GO TO 63
      RK1=GK
  07  CONTINUE
      GO TO 64
C LOOP FOR CALCULATING SUM OF X(I)**K1
  63  RK1=GK
  64  SUM00=0.0000
      DO 9 IH=1,IQ
      SUM00=SUM00+X(IH)**RK1
  09  CONTINUE
C ESTIMATE THETA (ETHETA)
C IF USING TEST ONE USE FOLLOWING THETA ESTIMATE
      ETHETA=(SUM00/IQ)**(1/RK1)
C IF USING TEST 2 OR 3 THE FOLLOWING ETHETA IS THE ESTIMATE OF S
      ETHETA=SUM00/IQ
```

```
C CALCULATE TEST STATISTIC (Z)
C IF USING TEST ONE USE FOLLOWING STATISTIC
      Z=(THETA1-THETA0)*(RK1**2)*IQ*(ETHETA-.5*(THETA0+THETA1))
C IF USING TEST TWO USE FOLLOWING STATISTIC
      Z=((THETA1**RK1)-(THETA0**RK1))*IQ*(ETHETA-.5*((THETA0**RK1)
     C+(THETA1**RK1)))/((ETHETA)**2.)
C IF USING TEST THREE USE FOLLOWING STATISTIC
      Z=IQ*RK1*ALOG(THETA0/THETA1)+((1./THETA0**RK1)-(1./THETA1
     C**RK1))*SUM1
C CALCULATE BOUNDARY COEFFICIENT    (C)
C IF USING TEST ONE USE FOLLOWING COEFFICIENT
      C=1.295466348
C IF USING TEST TWO OR THREE USE FOLLOWING COEFICIENT
      TERM1=(ALOG(ETHETA))**2.+.84556857*ALOG(ETHETA)
      TERM2=1.-((TERM1+.178746594)/(TERM1+1.82368056609))
      C=1./TERM2
C CALCULATE BOUNDARIES  (ACPT/RJCT)
      ACPT=C*B
      RJCT=C*A
C  CALCULATE TRUNCATION POINT IF TRUNCATING ON A MULTIPLE OF E(N)
      NTRUNC=RNUME/(RK1*ALOG(THETA1/THETA0)-((THETA1/THETA0)**RK1
     C)+1)+1
      NTRUNC=MULT*NTRUNC
C CONDUCT TEST
      IF(Z.LT.ACPT)GO TO 81
      IF(Z.GT.RJCT)GO TO 82
      IF((IQ).GE.NTRUNC)GO TO 83
   80 IQ=IQ+1
C OBTAIN ONE MORE FAILURE TIME AND ITERATE
      FX(IQ)=GGUBF(ISEED)
      X(IQ)=THETA*((-ALOG(1.-FX(IQ)))**RK0INV)
      GO TO 41
C INCREMENT ACCEPTANCE COUNTER
   81 NRACPT=NRACPT+1
      NROBSA(IA,IV)=NROBSA(IA,IV)+IQ
      GO TO 303
C INCREMENT REJECTION COUNTER
   82 NRRJCT=NRRJCT+1
      NROBSR(IA,IV)=NROBSR(IA,IV)+IQ
      GO TO 303
C TRUNCATION TEST
   83 TRUNCV=(ACPT+RJCT)/2.
      IF(Z.LT.TRUNCV)GO TO 84
      NTRUNRJ(IA,IV)=NTRUNRJ(IA,IV)+1
      GO TO 82
   84 NTRUNAC(IA,IV)=NTRUNAC(IA,IV)+1
      GO TO 81
C TAKE ANOTHER SAMPLE
C COUNT BAD OBS AND REPLACE SAMPLE
      GO TO 61
      GO TO 61
   61 NUMOBO(IA,IV)=NUMOBO(IA,IV)+1
```

```
 99   IF(NUMDBO(IA,IV).GE.30) GO TO 101
      GO TO 23
C    CALCULATE MEAN AND VARIANCE OF  K,THETA,AND ASN ESTIMATES
 303 RKK(IW)=RK1
      RNTRNG(IW)=NTRUNG/MULT
C IF USING TEST 1 USE FOLLOWING TO FIND MEAN OF THETA ESTIMATE
      EETHETA(IW)=ETHETA
C IF USING TEST 2 OR 3 USE FOLLOWING TO GET MEAN THETA ESTIMATE
      EETHETA(IW)=ETHETA**(1/RK1)
      RNSAMP=NSAMP
 03   CONTINUE
      SUMN=0.0
      SUMT=0.0
      SUMK=0.0
      DO 13 IWW=1,NSAMP
      SUMN=SUMN+RNTRNG(IWW)
      SUMK=RKK(IWW)+SUMK
      SUMT=EETHETA(IWW)+SUMT
 13   CONTINUE
      RNTRMN(IA,IV)=SUMN/NSAMP
      EKMEAN(IA,IV)=SUMK/NSAMP
      THETAM(IA,IV)=SUMT/NSAMP
      SUMVN=0.
      SUMVK=0.
      SUMVT=0.
      DO 14 IM=1,NSAMP
      RNTRNG(IM)=(RNTRNG(IM)-RNTRMN(IA,IV))**2
      RKK(IM)=(RKK(IM)-EKMEAN(IA,IV))**2
      EETHETA(IM)=(EETHETA(IM)-THETAM(IA,IV))**2
      SUMVN=SUMVN+RNTRNG(IM)
      SUMVK=SUMVK+RKK(IM)
      SUMVT=SUMVT+EETHETA(IM)
 14   CONTINUE
      VARN(IA,IV)=SUMVN/(NSAMP-1.)
      VARK(IA,IV)=SUMVK/(NSAMP-1.)
      VART(IA,IV)=SUMVT/(NSAMP-1.)
C FIND ASN, ALPHA, AND  BETA ERRORS FOR EACH RUN
C     CONVERT INTEGERS TO REAL NUMBERS
      RNRACPT=NRACPT
      RNRRJCT=NRRJCT
      RNROBSA(IA,IV)=NROBSA(IA,IV)
      RNROBSR(IA,IV)=NROBSR(IA,IV)
      OBSAVGA=RNROBSA(IA,IV)/RNRACPT
      IF(IV.EQ.2)GO TO 91
      RALPHA(IA)=RNRRJCT/RNSAMP
 91   ASNA(IA,IV)=OBSAVGA
      OBSAVGR=RNROBSR(IA,IV)/RNRRJCT
      IF(IV.EQ.1)GO TO 92
      RBETA(IA)=RNRACPT/RNSAMP
 92   ASNR(IA,IV)=OBSAVGR
 02   CONTINUE
 01   CONTINUE
```

```
C PRINT RESULTS
      PRINT*,"        SHAPE PARAMETER(K) =  ",RK0
C OUTPUT DATA
  95  PRINT 96
  96  FORMAT(1H ,7X,"INPUT              OUTPUT      ****ASN****
     CTRUNCATION                                 ESTIMATES")
      PRINT 97
  97  FORMAT(1H ,2X,"THETA  K0 ALPHA  ALPHA BETA      H0        H1
     C      H0       H1         K0           THETA0        K1         THE
     CTA1")
      PRINT 98
  98  FORMAT(1H ," H0  H1      &BETA          ACPT  RJCR  ACPT  RJCT
     CACC REJ ACC REJ     MEAN  VAR     MEAN  VAR    MEAN  VAR  MEAN
     CVAR")
      DO 12 IA=IA1,IA2,IA3
      PRINT 85,THETA0,THETA1,RK0,ALPHA(IA),RALPHA(IA),RBETA(IA),
     8ASNA(IA,1),ASNR(IA,1),ASNA(IA,2),ASNR(IA,2),NTRUNAC(IA,1),
     9NTRUNRJ(IA,1),NTRUNAC(IA,2),NTRUNRJ(IA,2),EKMEAN(IA,1),VARK
     C(IA,1),THETAM(IA,1),VART(IA,1),EKMEAN(IA,2),VARK(IA,2)
     C,THETAM(IA,2),VART(IA,2)
  85  FORMAT(1H ,F3.1,1X,F3.1,1X,F4.2,2X,F3.2,2X,F4.3,2X,F4.3,1X,
     CF5.1,1X,F5.1,1X,F5.1,1X,F5.1,1X,I3,1X,I3,1X,I3,1X,I3,3X,F5.
     C2,2X,F5.2,2X,F5.2,2X,F4.1,2X,F5.2,2X,F4.1,2X,F5.2,2X,F6.2)
  12  CONTINUE
      PRINT*," TRUNCATION POINT= ",MULT," *E(N)"
      PRINT 200
 200  FORMAT(1H,2X,"MEAN/VAR OF E(N)                 H0                    H1
     C                 ALPHA")
      DO 113 IA=IA1,IA2,IA3
      PRINT 62,RNTRMN(IA,1),VARN(IA,1),RNTRMN(IA,2),VARN(IA,2),
     CALPHA(IA)
  62  FORMAT(1H,25X,F5.1,2X,F7.2,4X,F5.1,2X,F6.2,14X,F3.2)
 113  CONTINUE
 111  CONTINUE
 101  STOP
      END
```

Appendix B

Output Risk and Average Sample Number for Truncation Point of 400

Risks - desired alpha and beta errors.

A_0 - average sample number to accept H_0 when H_0 is true.

R_0 - average sample number to reject H_0 when H_0 is true.

A_1 - average sample number to accept H_0 when H_1 is true.

R_1 - average sample number to reject H_0 when H_1 is true.

Numbers in parentheses behind average sample numbers are the number of tests resulting in truncation.

E(n) - expected sample number using input parameter values.

Tables are based upon 500 runs.

Table B-II-4 uses an input beta risk equal to twice the alpha risk.

Table B-I-1

Output Risks and Average Sample Number, Test One

$\theta_0=1.0$, $\theta_1=1.5$ Minimum Sample Size=5   Asymptotic θ Statistic

K	Risks	Alpha	Beta	A_0	R_0	A_1	R_1	E(n)
.5	.20	.340	.188	31.4	18.7	25.8	21.6	37.8
	.15	.396	.184	46.0	19.9	38.4	31.3	55.2
	.10	.236	.166	66.4	29.1	62.0	45.8	79.9
	.05	.206	.094	101.3(1)	33.2(1)	106.5(3)	78.2(5)	120.4
.75	.20	.288	.288	14.8	8.7	14.5	11.2	16.2
	.15	.268	.222	18.1	12.5	19.3	15.1	23.7
	.10	.242	.194	27.5	15.0	18.8	20.4	34.3
	.05	.212	.102	41.8	17.7	31.8	36.2	51.7
1.0	.20	.232	.300	9.4	7.8	8.5	9.1	8.8
	.15	.236	.206	11.3	9.0	10.8	10.7	12.8
	.10	.212	.226	16.6	10.1	16.3	12.6	18.6
	.05	.148	.134	23.1	11.9	14.3	20.6	28.0
1.5	.20	.150	.246	6.3	7.1	6.9	6.5	3.6
	.15	.154	.238	7.5	6.3	7.5	7.6	5.3
	.10	.130	.182	8.1	7.4	10.4	8.2	7.7
	.05	.124	.178	10.8	7.8	11.0	12.1	11.6
2.0	.20	.096	.212	5.6	5.6	6.0	5.8	1.9
	.15	.096	.180	5.8	5.4	6.7	6.3	2.8
	.10	.078	.154	6.8	5.6	6.2	7.0	4.0
	.05	.066	.120	7.4	6.9	8.0	8.7	6.0
3.0	.20	.038	.146	5.1	5.3	5.6	5.3	.7
	.15	.024	.118	5.2	5.4	6.0	5.6	1.0
	.10	.032	.114	5.2	6.1	5.3	5.6	1.5
	.05	.020	.104	5.4	6.2	6.4	6.2	2.3

Table B-I-2

Output Risks and Average Sample Number, Test One

$\theta_0=1$, $\theta_1=2$ Minimum Sample Size=3 Asymptotic θ Statistic

K	Risks	Alpha	Beta	A_0	R_0	A_1	R_1	E(n)
.5	.20	.284	.338	7.6	5.3	6.3	5.8	12.3
	.15	.292	.322	8.4	5.2	8.5	7.4	18.0
	.10	.270	.278	12.4	7.2	10.6	10.6	26.0
	.05	.240	.200	19.2	8.0	16.0	14.4	39.2
.75	.20	.220	.310	4.5	3.9	4.3	4.3	5.1
	.15	.254	.318	5.1	3.9	5.1	5.1	7.5
	.10	.218	.294	6.6	4.5	6.2	6.0	10.9
	.05	.182	.264	8.2	5.3	8.4	8.3	16.4
1.0	.20	.152	.248	3.6	3.9	4.0	3.7	2.7
	.15	.156	.294	4.2	3.6	3.9	4.1	4.0
	.10	.140	.318	4.5	4.0	4.4	4.3	5.7
	.05	.138	.258	5.8	4.2	5.7	5.6	8.6
1.5	.20	.110	.258	3.2	3.3	3.3	3.4	1.1
	.15	.092	.272	3.3	3.4	3.6	3.4	1.5
	.10	.076	.220	3.5	3.4	3.7	3.7	2.2
	.05	.056	.198	3.9	3.3	4.2	4.1	3.4
2.0	.20	.042	.184	3.1	3.0	3.4	3.2	.5
	.15	.038	.160	3.1	3.3	3.4	3.3	.8
	.10	.042	.184	3.2	3.1	3.6	3.4	1.1
	.05	.018	.168	3.4	3.3	3.5	3.6	1.6
3.0	.20	.004	.096	3.0	3.0	3.2	3.1	.2
	.15	.000	.116	3.0	--	3.2	3.1	.2
	.10	.002	.092	3.0	3.0	3.2	3.1	.4
	.05	.000	.116	3.0	--	3.2	3.2	.5

Table B-I-3

Output Risks and Average Sample Number, Test Two

$\theta_0=1$, $\theta_1=2$ Minimum Sample Size=5 Asymptotic θ Statistic

K	Risks	Alpha	Beta	A_0	R_0	A_1	R_1	E(n)
.5	.20	.292	.328	9.5	7.6	8.6	7.8	12.3
	.15	.252	.268	11.3	7.9	11.7	9.5	18.0
	.10	.260	.246	14.6	8.2	14.7	12.3	26.0
	.05	.216	.164	21.9	11.1	21.6	17.1	39.2
.75	.20	.200	.294	6.3	6.1	6.6	6.1	5.1
	.15	.140	.320	7.3	6.5	7.3	7.3	7.5
	.10	.174	.256	8.3	7.1	9.0	7.7	10.9
	.05	.126	.202	10.3	8.8	9.9	9.6	16.4
1.0	.20	.142	.238	5.6	5.4	6.1	5.7	2.7
	.15	.124	.242	5.8	5.7	6.5	6.0	4.0
	.10	.114	.220	6.4	5.7	7.4	6.7	5.7
	.05	.116	.184	7.2	5.7	8.0	7.5	8.6
1.5	.20	.046	.176	5.1	5.0	5.3	5.3	1.1
	.15	.052	.180	5.2	5.6	5.6	5.4	1.5
	.10	.056	.128	5.2	5.6	5.7	5.6	2.2
	.05	.026	.192	5.5	5.3	5.8	6.1	3.4
2.0	.20	.012	.126	5.0	5.5	5.3	5.1	.5
	.15	.006	.122	5.0	5.0	5.6	5.2	.8
	.10	.012	.106	5.1	5.2	5.5	5.3	1.1
	.05	.014	.106	5.1	5.3	5.8	5.5	1.6
3.0	.20	.000	.092	5.0	--	5.2	5.0	.2
	.15	.000	.042	5.0	--	5.0	5.1	.2
	.10	.000	.062	5.0	--	5.1	5.1	.4
	.05	.000	.044	5.0	--	5.2	5.2	.5

Table B-I-4

<u>Output Risks and Average Sample Number</u>, <u>Test One</u>, <u>Beta=2*Alpha</u>

$\theta_0=1$, $\theta_1=2$ Minimum Sample Size=5 Asymptotic θ Statistic

K	Risks	Alpha	Beta	A_0	R_0	A_1	R_1
.5	.20/.40	.260	.374	6.2	5.9	6.6	6.3
	.15/.30	.209	.392	7.8	7.0	9.1	7.7
	.10/.20	.268	.322	10.3	8.1	10.6	9.6
	.05/.10	.244	.218	16.4	10.5	15.4	14.8
.75	.20/.40	.162	.364	5.4	5.7	5.8	5.6
	.15/.30	.178	.330	5.8	5.4	6.1	6.0
	.10/.20	.150	.288	6.7	6.4	7.5	6.8
	.05/.10	.150	.288	8.6	6.4	9.7	9.2
1.0	.20/.40	.112	.270	5.1	5.4	5.6	5.2
	.15/.30	.132	.320	5.4	5.6	5.7	5.6
	.10/.20	.104	.232	5.8	6.3	6.4	6.1
	.05/.10	.096	.244	6.5	7.2	7.5	6.9
1.5	.20/.40	.066	.178	5.0	5.2	5.2	5.2
	.15/.30	.042	.174	5.1	5.3	5.2	5.2
	.10/.20	.024	.202	5.1	5.8	5.7	5.5
	.05/.10	.028	.182	5.3	5.3	6.0	5.8
2.0	.20/.40	.018	.140	5.0	5.2	5.3	5.1
	.15/.30	.012	.118	5.0	5.2	5.2	5.1
	.10/.20	.014	.098	5.0	5.1	5.9	5.1
	.05/.10	.016	.134	5.1	5.3	5.6	5.4
3.0	.20/.40	.000	.066	5.0	--	5.0	5.0
	.15/.30	.000	.062	5.0	--	5.3	5.0
	.10/.20	.000	.056	5.0	--	5.2	5.0
	.05/.10	.000	.048	5.0	--	5.2	5.1

Table B-II-1

Output Risks and Average Sample Number, Test Two

$\theta_0=1$, $\theta_1=1.5$ Minimum Sample Size=5 Asymptotic G Statistic

K	Risks	Alpha	Beta	A_0	R_0	A_1	R_1	E(n)
.5	.20	.062	.478	32.9	76.3	26.0	77.0	37.8
	.15	.044	.360	39.6	104.5	24.0	110.3(6)	55.2
	.10	.052	.318	55.5	187.5	38.0	143.6(2)	79.9
	.05	.010	.258	81.5	348.0	44.1(2)	206.6(3)	120.4
.75	.20	.052	.396	18.5	47.1	13.8	45.2	16.2
	.15	.038	.422	22.5	52.3	16.2	57.6	23.7
	.10	.018	.312	26.6	68.2	16.5	78.8	34.3
	.05	.000	.236	35.6	--	18.2	112.0	51.7
1.0	.20	.028	.394	12.0	31.9	10.8	32.0	8.8
	.15	.010	.364	13.7	45.8	11.0	41.9	12.8
	.10	.006	.322	16.4	39.7	13.5	56.4	18.6
	.05	.002	.260	20.3	70.0	10.7	78.9	28.0
1.5	.20	.010	.324	7.5	21.0	8.7	23.3	3.6
	.15	.006	.310	8.7	23.7	9.7	28.8	5.3
	.10	.000	.264	8.4	--	9.8	37.1	7.7
	.05	.000	.224	11.2	--	9.1	50.6	11.6
2.0	.20	.000	.280	6.0	--	7.4	19.3	1.9
	.15	.000	.234	6.4	--	8.4	24.1	2.8
	.10	.000	.258	6.8	--	9.3	29.6	4.0
	.05	.000	.156	6.8	--	7.4	40.5	6.0
3.0	.20	.000	.224	5.1	--	7.1	17.6	.7
	.15	.000	.174	5.2	--	6.6	21.8	1.0
	.10	.000	.168	5.2	--	6.8	26.6	1.5
	.05	.000	.152	5.3	--	7.7	35.1	2.3

Table B-II-2

Output Risks and Average Sample Number, Test Two

$\theta_0=1, \theta_1=1.5$ Minimum Sample Size=10 Asymptotic G Statistic

K	Risks	Alpha	Beta	A_0	R_0	A_1	R_1	E(n)
.5	.20	.082	.374	38.5	89.3	33.7	79.1	37.8
	.15	.024	.312	50.2	99.9	39.7(1)	107.3(1)	55.2
	.10	.030	.264	72.5(1)	184.4(1)	54.0(2)	150.8(10)	79.9
	.05	.036	.176	102.7(12)	349.2(11)	75.5(7)	197.3(21)	120.4
.75	.20	.060	.326	21.0	51.8	22.8	47.4	16.2
	.15	.032	.256	26.1	55.6	28.3	60.3	23.7
	.10	.022	.270	33.2	95.5	23.4	82.4	34.3
	.05	.004	.170	41.3(1)	70.0	37.4(1)	115.4(1)	51.7
1.0	.20	.054	.278	15.4	32.4	18.8	32.2	8.8
	.15	.026	.296	17.8	37.5	18.3	41.9	12.8
	.10	.008	.232	21.3	44.5	23.1	57.7	18.6
	.05	.000	.168	24.7	--	24.2	76.8	28.0
1.5	.20	.018	.220	11.8	20.4	13.4	22.9	3.6
	.15	.004	.236	12.9	31.5	15.0	29.1	5.3
	.10	.002	.192	13.5	48.0	16.1	36.9	7.7
	.05	.000	.160	14.7	--	15.9	50.4	11.6
2.0	.20	.002	.172	10.6	15.0	12.9	19.4	1.9
	.15	.002	.132	10.7	25.0	12.0	25.0	2.8
	.10	.000	.138	11.0	--	12.8	30.8	4.0
	.05	.000	.112	11.2	--	13.9	40.3	6.0
3.0	.20	.000	.108	10.0	--	12.1	17.6	.7
	.15	.000	.094	10.0	--	12.1	21.9	1.0
	.10	.000	.078	10.1	--	13.2	26.6	1.5
	.05	.000	.058	10.1	--	11.9	34.9	2.3

Table B-II-3

Output Risks and Average Sample Number, Test Two

$\theta_0=1$, $\theta_1=2.0$ Minimum Sample Size=5   Asymptotic G Statistic

K	Risks	Alpha	Beta	A_0	R_0	A_1	R_1	E(n)
.5	.20	.036	.398	14.7	37.3	12.8	38.3	12.3
	.15	.026	.386	15.9	56.9	17.6	52.2	18.0
	.10	.012	.308	22.3	60.5	17.5	67.7	26.0
	.05	.000	.276	26.8	--	13.7	92.8	39.2
.75	.20	.018	.368	8.6	21.2	9.4	26.8	5.1
	.15	.006	.350	10.3	39.7	10.0	32.1	7.5
	.10	.002	.300	10.7	36.0	11.4	42.9	10.9
	.05	.000	.226	13.9	--	11.3	58.7	16.4
1.0	.20	.006	.312	6.9	19.7	7.8	20.6	2.7
	.15	.000	.252	7.7	--	8.9	26.9	4.0
	.10	.000	.234	8.0	--	7.7	33.6	5.7
	.05	.000	.216	8.8	--	9.6	44.2	8.6
1.5	.20	.000	.250	5.5	--	6.6	18.1	1.1
	.15	.000	.222	5.3	--	7.9	22.1	1.5
	.10	.000	.208	5.5	--	7.6	27.1	2.2
	.05	.000	.156	5.8	--	6.4	35.6	3.4
2.0	.20	.000	.174	5.1	--	6.3	17.6	.5
	.15	.000	.172	5.1	--	6.4	21.6	.8
	.10	.000	.156	5.1	--	6.6	27.0	1.1
	.05	.000	.138	5.1	--	6.6	34.5	1.6
3.0	.20	.000	.108	5.0	--	6.8	21.0	.2
	.15	.000	.102	5.0	--	6.1	25.2	.2
	.10	.000	.094	5.0	--	5.9	31.3	.4
	.05	.000	.082	5.0	--	6.0	40.3	.5

Table B-II-4

Output Risks and Average Sample Number, Test Two

$\theta_0=1$, $\theta_1=2.0$ Minimum Sample Size=10 Asymptotic G Statistic

K	Risks	Alpha	Beta	A_0	R_0	A_1	R_1	E(n)
.5	.20	.038	.332	18.6	36.9	19.6	37.1	12.3
	.15	.028	.286	23.7	43.9	24.2	52.1	18.0
	.10	.014	.242	24.3	83.3	23.1	68.8	26.0
	.05	.004	.188	34.0	73.0	29.1	97.3	39.2
.75	.20	.020	.242	12.9	24.6	15.6	25.8	5.1
	.15	.008	.226	13.6	36.5	17.7	34.3	7.5
	.10	.002	.182	15.4	52.0	17.6	43.9	10.9
	.05	.002	.182	17.2	58.0	18.3	58.1	16.4
1.0	.20	.008	.200	11.2	17.5	13.6	21.5	2.7
	.15	.000	.150	11.5	--	13.4	26.7	4.0
	.10	.002	.150	12.3	47.0	15.4	33.2	5.7
	.05	.000	.116	13.1	--	15.4	46.3	8.6
1.5	.20	.002	.114	10.2	16.0	12.3	17.9	1.1
	.15	.000	.104	10.2	--	12.8	22.5	1.5
	.10	.000	.116	10.2	--	13.5	27.5	2.2
	.05	.000	.072	10.2	--	14.9	35.9	3.4
2.0	.20	.000	.096	10.0	--	11.3	18.0	.5
	.15	.000	.084	10.0	--	12.9	21.6	.8
	.10	.000	.072	10.0	--	11.7	26.5	1.1
	.05	.000	.058	10.0	--	11.1	34.7	1.6
3.0	.20	.000	.040	10.0	--	11.3	20.9	.2
	.15	.000	.028	10.0	--	11.8	25.2	.2
	.10	.000	.026	10.0	--	11.8	31.3	.4
	.05	.000	.032	10.0	--	13.3	40.3	.5

Table B-III-1

<u>Output Risks and Average Sample Number, Test Three</u>

$\theta_0=1$, $\theta_1=1.5$ Minimum Sample Number=5 Exact G. Statistic

K	Risk	Alpha	Beta	A_0	R_0	A_1	R_1	E(n)
.5	.20	.228	.236	46.9	29.4	36.7	44.6	37.8
	.15	.146	.152	67.9(1)	55.4	59.2	68.7(1)	55.2
	.10	.124	.118	97.9(3)	50.8	102.1(3)	87.6(2)	79.9
	.05	.074	.036	138.1(15)	111.1(2)	205.9(5)	142.7(18)	120.4
.75	.20	.236	.230	22.0	20.0	17.8	22.6	16.2
	.15	.136	.158	30.9	18.7	28.4	30.4	23.7
	.10	.136	.128	47.9	23.8	37.3	43.0	34.3
	.05	.076	.068	66.3	40.1	45.2	68.7(1)	51.7
1.0	.20	.192	.200	13.9	12.0	12.8	15.5	8.8
	.15	.160	.170	19.9	13.2	14.6	18.4	12.8
	.10	.132	.128	27.0	15.1	19.8	26.4	18.6
	.05	.096	.078	37.0	16.1	24.7	38.7	28.0
1.5	.20	.124	.156	8.5	8.2	8.8	9.2	3.6
	.15	.112	.138	9.7	10.5	7.4	10.8	5.3
	.10	.092	.132	13.0	8.3	11.3	14.5	7.7
	.05	.078	.050	17.7	12.9	8.0	19.5	11.6
2.0	.20	.118	.122	6.7	6.5	7.1	6.7	1.9
	.15	.080	.118	7.8	6.8	7.7	8.1	2.8
	.10	.084	.072	9.0	7.1	6.8	9.3	4.0
	.05	.054	.060	12.0	7.9	9.6	12.2	6.0
3.0	.20	.058	.080	5.4	5.5	6.0	5.7	.7
	.15	.050	.044	5.9	6.8	6.0	6.1	1.0
	.10	.030	.036	6.2	6.3	5.8	6.5	1.5
	.05	.034	.032	7.1	7.2	7.1	7.3	2.3

Table B-III-2

Output Risk and Average Sample Number, Test Three

$\theta_0=1$, $\theta_1=2$ Minimum Sample Number=3 Exact G Statistic

K	Risk	Alpha	Beta	A_0	R_0	A_1	R_1	E(n)
.5	.20	.250	.260	16.3	10.1	11.4	15.6	12.3
	.15	.202	.212	24.2	11.7	17.1	20.6	18.0
	.10	.166	.144	32.9	13.8	27.1	31.2	26.0
	.05	.082	.082	50.3	12.4	33.5	45.8	39.2
.75	.20	.182	.234	8.3	6.2	7.4	8.3	5.1
	.15	.172	.188	11.3	7.5	8.4	11.1	7.5
	.10	.126	.132	14.8	7.0	10.4	14.9	10.9
	.05	.082	.086	22.1	8.6	12.1	23.6	16.4
1.0	.20	.176	.214	5.9	4.8	5.1	5.5	2.7
	.15	.140	.156	7.6	5.8	6.4	7.7	4.0
	.10	.138	.114	9.4	5.9	6.4	9.5	5.7
	.05	.088	.118	13.7	6.2	8.8	13.9	8.6
1.5	.20	.122	.152	4.4	3.8	3.7	4.2	1.1
	.15	.112	.084	5.0	4.1	4.5	4.9	1.5
	.10	.078	.114	5.5	3.9	3.8	6.0	2.2
	.05	.058	.052	7.4	4.9	4.4	7.6	3.4
2.0	.20	.098	.068	3.7	3.3	3.4	3.8	.5
	.15	.072	.078	4.0	3.4	3.6	4.0	.8
	.10	.076	.064	4.3	3.5	3.6	4.4	1.1
	.05	.054	.046	5.2	3.5	3.9	5.1	1.6
3.0	.20	.052	.042	3.2	3.1	3.6	3.3	.2
	.15	.042	.024	3.3	3.2	3.2	3.3	.2
	.10	.030	.022	3.5	3.1	3.6	3.5	.4
	.05	.038	.014	3.7	3.4	3.0	3.8	.5

Table B-III-3

Output Risk and Average Sample Number, Test Three

$\theta_0=1$, $\theta_1=2.0$ Minimum Sample Size=5   Exact G Statistic

K	Risk	Alpha	Beta	A_0	R_0	A_1	R_1	E(n)
.5	.20	.186	.188	17.8	13.5	16.2	18.2	12.3
	.15	.168	.162	24.0	19.3	18.9	24.8	18.0
	.10	.126	.114	35.2	18.3	26.1	36.2	26.0
	.05	.078	.044	54.0	30.7	27.9	54.1	39.2
.75	.20	.154	.188	9.9	9.2	9.7	10.1	5.1
	.15	.156	.180	12.3	9.8	10.9	13.0	7.5
	.10	.130	.112	17.5	11.3	13.4	18.0	10.9
	.05	.060	.100	22.8	16.7	13.0	25.4	16.4
1.0	.20	.138	.152	7.3	8.2	7.9	7.9	2.7
	.15	.114	.114	8.7	8.0	7.9	9.4	4.0
	.10	.074	.126	11.6	7.9	9.4	11.8	5.7
	.05	.064	.060	15.3	10.1	10.8	15.7	8.6
1.5	.20	.066	.062	5.8	7.2	6.5	6.1	1.1
	.15	.056	.070	6.4	6.7	6.2	6.6	1.5
	.10	.066	.058	6.8	6.6	7.0	7.4	2.2
	.05	.026	.038	8.1	7.1	6.2	9.0	3.4
2.0	.20	.036	.050	5.4	5.2	5.5	5.5	.5
	.15	.034	.030	5.6	5.5	6.5	5.6	.8
	.10	.026	.028	5.7	5.8	5.6	6.1	1.1
	.05	.018	.014	6.6	5.0	6.6	6.7	1.6
3.0	.20	.018	.006	5.1	6.1	5.3	5.1	.2
	.15	.018	.006	5.1	5.4	6.0	5.1	.2
	.10	.012	.008	5.2	6.2	5.0	5.2	.4
	.05	.004	.000	5.4	7.0	--	5.4	.5

Appendix C

Output Risk and Average Sample Number For Truncation Point of $2 \cdot E(n)$

$E(n)_0$ - mean of the expected sample for each test at termination when input $\theta=\theta_0$ using estimated parameter values.

$E(n)_1$ - mean of the expected sample numbers for each test at termination when $\theta=\theta_1$ using estimated parameter values.

Other column headings are the same as Appendix B. Tables are based upon five hundred Monte Carlo repetitions.

Table C-I-1

Output Risk and Average Sample Number, Test One
$\theta_0=1$, $\theta_1=1.5$ Minimum Sample Size=5 Asymptotic θ Statistic

K	Risk	Alpha	Beta	A_0	R_0	A_1	R_1	$E(n)_0$	$E(n)_1$
.5	.20	.332	.262	29.7(17)	17.3(9)	24.2(10)	19.9(13)	29.2	27.2
	.15	.354	.202	41.5(10)	18.1(6)	43.6(12)	28.7(22)	42.5	39.8
	.10	.266	.136	59.0(6)	26.3(8)	61.7(8)	44.1(17)	64.8	61.4
	.05	.218	.106	94.6(17)	41.5(6)	130.9(14)	72.4(19)	103.0	95.8
.75	.20	.288	.280	13.2(25)	10.2(17)	13.1(21)	11.0(27)	11.9	11.0
	.15	.266	.228	17.6(26)	13.2(17)	17.1(15)	13.9(27)	17.1	15.9
	.10	.214	.164	25.9(15)	13.3(7)	29.5(12)	19.9(20)	26.4	24.7
	.05	.190	.172	38.6(17)	18.1(6)	30.6(12)	30.8(18)	41.3	37.3
1.0	.20	.218	.310	8.2(46)	7.4(29)	8.9(38)	7.7(36)	6.6	6.3
	.15	.212	.262	10.4(44)	8.9(18)	10.5(24)	10.0(37)	9.5	9.1
	.10	.206	.266	14.3(27)	9.1(17)	14.1(22)	13.1(35)	13.7	12.6
	.05	.146	.162	21.1(24)	13.6(17)	21.6(21)	19.0(31)	21.4	20.0
1.5	.20	.204	.288	5.8(67)	5.6(45)	6.3(44)	5.8(64)	3.2	3.0
	.15	.170	.270	6.5(58)	5.7(30)	6.9(42)	6.4(41)	4.4	4.0
	.10	.144	.230	7.7(54)	6.9(27)	9.2(36)	7.6(55)	6.2	5.7
	.05	.130	.188	9.4(39)	8.9(27)	10.1(21)	10.3(51)	8.8	8.9
2.0	.20	.130	.274	5.2(55)	5.1(30)	5.5(52)	5.3(54)	2.1	1.9
	.15	.114	.234	5.5(59)	5.5(31)	5.9(51)	5.5(66)	2.7	2.4
	.10	.130	.228	6.0(70)	6.0(33)	6.1(51)	6.0(65)	3.5	3.1
	.05	.102	.168	6.7(54)	6.0(30)	7.3(30)	6.9(58)	4.8	4.6
3.0	.20	.064	.148	5.0(17)	5.0(10)	5.1(31)	5.0(45)	1.2	1.2
	.15	.032	.164	5.0(37)	5.1(11)	5.2(42)	5.1(59)	1.5	1.4
	.10	.034	.138	5.1(35)	5.1(12)	5.4(36)	5.2(46)	1.8	1.7
	.05	.048	.140	5.2(44)	5.3(17)	5.9(36)	5.5(78)	2.5	2.2

Table C-I-2

Output Risk and Average Sample Number, Test One
$\theta_0=1$, $\theta_1=2$ Minimum Sample Number=5 Asymptotic θ Statistic

k	Risk	Alpha	Beta	A_0	R_0	A_1	R_1	$E(n)_0$	$E(n)_1$
.5	.20	.266	.322	8.9(16)	6.8(11)	9.4(12)	7.5(14)	9.2	9.1
	.15	.288	.282	11.3(7)	7.7(6)	11.1(5)	9.4(15)	12.8	12.4
	.10	.228	.248	14.7(2)	8.4	16.6(12)	12.0(8)	19.2	18.3
	.05	.222	.200	21.2(5)	11.0(4)	23.4(3)	16.8(4)	28.7	28.2
.75	.20	.210	.298	5.9(31)	6.0(18)	6.8(29)	6.1(36)	4.3	4.3
	.15	.170	.296	6.7(31)	5.9(16)	7.4(26)	6.4(16)	5.8	5.5
	.10	.164	.288	7.9(20)	7.2(11)	7.6(13)	7.6(20)	8.2	7.7
	.05	.150	.216	9.9(17)	7.5(9)	12.6(16)	9.3(17)	11.8	11.6
1.0	.20	.158	.300	5.3(36)	5.2(22)	5.6(40)	5.3(35)	2.7	2.5
	.15	.116	.260	5.7(43)	5.3(11)	6.2(36)	5.7(34)	3.7	3.3
	.10	.142	.248	6.1(33)	5.8(17)	7.0(24)	6.3(27)	4.9	4.9
	.05	.084	.214	7.1(22)	6.0(10)	7.8(19)	7.3(31)	7.1	6.8
1.5	.20	.072	.200	5.0(19)	5.0(12)	5.2(31)	5.0(38)	1.4	1.4
	.15	.040	.248	5.0(24)	5.0(8)	5.3(34)	5.1(37)	1.8	1.7
	.10	.040	.170	5.1(26)	5.1(10)	5.4(28)	5.2(43)	2.4	2.1
	.05	.060	.170	5.4(27)	5.3(14)	5.9(30)	5.6(45)	3.2	2.9
2.0	.20	.026	.142	5.0(7)	5.0(4)	5.0(15)	5.0(27)	1.1	1.1
	.15	.016	.126	5.0(6)	5.0(3)	5.1(23)	5.0(25)	1.2	1.3
	.10	.020	.128	5.0(6)	5.0(4)	5.2(28)	5.0(38)	1.6	1.4
	.05	.022	.136	5.0(16)	5.2(4)	5.2(32)	5.2(37)	1.8	1.8
3.0	.20	.000	.056	5.0	--	5.0(1)	5.0(9)	1.0	1.0
	.15	.000	.082	5.0	--	5.0(16)	5.0(9)	1.0	1.0
	.10	.000	.076	5.0	--	5.0(14)	5.0(15)	1.0	1.1
	.05	.000	.056	5.0(1)	--	5.1(9)	5.0(28)	1.2	1.2

Table C-II-1

Output Risk and Average Sample Numbers, Test Two

$\theta_0=1$, $\theta_1=1.5$ Minimum Sample Size=5 Asymptotic G Statistic

K	Risk	Alpha	Beta	A_0	R_0	A_1	R_1	$E(n)_0$	$E(n)_1$
.5	.20	.134	.464	20.5(24)	55.3(58)	22.2(34)	62.9(110)	32.5	33.3
	.15	.112	.368	30.4(33)	87.0(43)	28.6(17)	86.0(111)	48.2	49.8
	.10	.050	.376	42.6(27)	123.0(18)	33.6(16)	121.3(88)	67.3	71.2
	.05	.038	.258	72.3(26)	205.0(17)	46.1(13)	175.7(88)	109.2	107.7
.75	.20	.198	.418	10.8(30)	18.6(97)	12.1(28)	25.2(280)	13.8	12.7
	.15	.134	.356	12.8(28)	31.6(67)	14.6(23)	40.1(267)	21.0	20.2
	.10	.088	.326	19.1(25)	45.7(44)	19.0(19)	58.6(206)	29.5	29.9
	.05	.034	.276	28.0(34)	60.2(17)	24.0(19)	88.9(166)	43.4	46.8
1.0	.20	.202	.384	7.1(32)	9.6(101)	8.2(33)	13.3(308)	7.5	7.1
	.15	.162	.350	9.5(47)	13.7(81)	9.4(25)	19.8(324)	10.6	10.1
	.10	.098	.334	11.4(31)	18.2(49)	15.2(37)	29.3(326)	15.6	15.0
	.05	.054	.310	16.7(38)	25.7(27)	15.9(25)	49.9(320)	23.7	23.6
1.5	.20	.138	.352	5.5(64)	5.9(69)	6.1(55)	6.8(324)	3.4	3.0
	.15	.124	.290	5.9(48)	6.8(62)	7.1(48)	8.7(355)	4.6	4.3
	.10	.150	.300	6.5(48)	7.4(75)	7.1(36)	11.2(350)	6.6	5.8
	.05	.084	.234	8.4(35)	11.3(42)	9.9(33)	17.7(383)	10.3	8.8
2.0	.20	.070	.270	5.1(55)	5.1(35)	5.3(46)	5.5(365)	2.1	1.9
	.15	.100	.240	5.3(71)	5.3(50)	5.5(41)	6.0(380)	2.9	2.4
	.10	.100	.284	5.4(61)	5.7(50)	6.2(48)	7.2(358)	3.6	3.2
	.05	.070	.220	5.9(53)	6.1(35)	7.1(41)	9.8(390)	5.5	4.4
3.0	.20	.014	.258	5.0(17)	5.0(7)	5.0(65)	5.0(371)	1.2	1.2
	.15	.008	.274	5.0(30)	5.3(4)	5.2(79)	5.1(363)	1.4	1.5
	.10	.008	.222	5.0(29)	5.3(4)	5.1(56)	5.2(389)	1.8	1.5
	.05	.014	.224	5.1(34)	5.3(7)	5.5(66)	5.8(388)	2.4	2.1

Table C-II-2

Output Risk and Average Sample Numbers, Test Two

$\theta_0=1$, $\theta_1=2$ Minimum Sample Size=5   Asymptotic G Statistic

K	Risk	Alpha	Beta	A_0	R_0	A_1	R_1	$E(n)_0$	$E(n)_1$
.5	.20	.266	.322	8.9(16)	6.8(11)	9.4(12)	7.5(14)	9.2	9.1
	.15	.288	.282	11.3(7)	7.7(6)	11.1(5)	9.4(15)	12.8	12.4
	.10	.228	.248	14.7(2)	8.4	16.6(12)	12.0(8)	19.2	18.3
	.05	.222	.200	21.2(5)	11.0(4)	23.4(3)	16.8(4)	28.7	28.2
.75	.20	.210	.298	5.9(31)	6.0(18)	6.8(29)	6.1(36)	4.3	4.3
	.15	.170	.296	6.7(31)	5.9(16)	7.4(26)	6.4(16)	5.8	5.5
	.10	.164	.288	7.9(20)	7.2(11)	7.6(13)	7.6(20)	8.2	7.7
	.05	.150	.216	9.9(17)	7.5(9)	12.6(16)	9.3(17)	11.8	11.6
1.0	.20	.158	.300	5.3(36)	5.2(22)	5.6(40)	5.3(35)	2.7	2.5
	.15	.116	.260	5.7(43)	5.3(11)	6.2(36)	5.7(34)	3.7	3.3
	.10	.142	.248	6.1(33)	5.8(17)	7.0(24)	6.3(27)	4.9	4.8
	.05	.084	.214	7.1(22)	6.0(10)	7.8(19)	7.3(31)	7.1	6.8
1.5	.20	.072	.200	5.0(19)	5.0(12)	5.2(31)	5.0(38)	1.4	1.4
	.15	.040	.248	5.0(24)	5.0(8)	5.0(34)	5.1(37)	1.8	1.7
	.10	.040	.170	5.1(26)	5.1(10)	5.4(28)	5.2(43)	2.4	2.1
	.05	.060	.170	5.4(27)	5.3(14)	5.9(30)	5.6(45)	3.2	2.9
2.0	.20	.026	.142	5.0(7)	5.0(4)	5.4(15)	5.0(27)	1.1	1.1
	.15	.016	.126	5.0(6)	5.0(3)	5.1(23)	5.0(25)	1.2	1.3
	.10	.020	.128	5.0(6)	5.0(4)	5.2(28)	5.0(38)	1.6	1.4
	.05	.022	.136	5.0(16)	5.2(4)	5.2(32)	5.2(37)	1.8	1.8
3.0	.20	.000	.056	5.0	--	5.0(1)	5.0(9)	1.0	1.0
	.15	.000	.082	5.0	--	5.0(16)	5.0(9)	1.0	1.0
	.10	.000	.076	5.0	--	5.0(14)	5.0(15)	1.0	1.1
	.05	.000	.056	5.0(1)	--	5.1(9)	5.0(28)	1.2	1.2

Table C-III-1

<u>Output Risks and Average Sample Number, Test Three</u>

$\theta_0=1$, $\theta_1=1.5$ Minimum Sample=5   Exact G Statistic

K	Risk	Alpha	Beta	A_0	R_0	A_1	R_1	$E(n)_0$	$E(n)_1$
.5	.20	.238	.240	36.8(52)	31.8(32)	46.8(45)	35.3(54)	31.3	30.1
	.15	.222	.212	56.3(56)	48.9(32)	69.7(38)	56.5(79)	48.8	46.5
	.10	.158	.134	84.4(65)	63.0(20)	116.8(30)	82.8(77)	72.0	70.1
	.05	.102	.076	127.4(62)	126.5(21)	158.9(17)	127.1(58)	114.9	108.5
.75	.20	.258	.258	15.8(75)	13.7(51)	18.5(44)	15.8(77)	12.4	11.7
	.15	.206	.182	23.9(68)	23.4(48)	24.2(31)	24.9(87)	19.2	18.0
	.10	.194	.138	38.3(90)	28.7(35)	28.3(23)	35.5(72)	29.0	27.4
	.05	.100	.106	55.3(76)	40.8(20)	69.6(21)	56.0(87)	45.2	43.2
1.0	.20	.258	.268	10.4(99)	8.9(62)	10.5(52)	10.1(107)	6.6	6.3
	.15	.234	.222	13.9(94)	11.1(50)	12.6(38)	14.6(117)	9.6	8.9
	.10	.188	.162	19.5(92)	16.6(51)	20.8(31)	19.7(82)	14.3	13.6
	.05	.132	.106	29.9(83)	24.7(33)	32.4(30)	31.3(90)	23.3	22.2
1.5	.20	.216	.228	6.2(112)	6.0(74)	7.0(61)	6.6(148)	3.1	3.0
	.15	.202	.216	7.8(116)	7.0(71)	7.8(62)	7.7(143)	4.5	3.8
	.10	.182	.150	9.6(112)	8.5(60)	8.8(38)	10.2(139)	6.0	5.5
	.05	.168	.136	13.7(117)	10.0(63)	15.4(40)	14.3(131)	8.8	8.5
2.0	.20	.164	.206	5.4(119)	5.1(51)	5.5(53)	5.4(134)	2.1	1.9
	.15	.198	.202	6.0(130)	5.3(75)	6.3(59)	6.0(153)	2.6	2.4
	.10	.166	.150	6.6(126)	5.8(70)	7.4(52)	6.9(163)	3.3	3.2
	.05	.146	.108	8.1(151)	6.5(65)	7.9(37)	8.8(198)	4.5	4.3
3.0	.20	.110	.124	5.0(73)	5.0(32)	5.0(48)	5.0(95)	1.2	1.2
	.15	.078	.110	5.0(111)	5.1(30)	5.3(37)	5.1(114)	1.4	1.4
	.10	.090	.096	5.2(120)	5.0(37)	5.7(34)	5.3(144)	1.7	1.7
	.05	.094	.088	5.6(138)	5.2(40)	5.8(36)	5.8(173)	2.2	2.2

Table C-III-2

Output Risk and Average Sample Numbers, Test Three

θ_0=1.0, θ_1=2 Minimum Sample Size=5 Exact G Statistic

K	Risk	Alpha	Beta	A_0	R_0	A_1	R_1	$E(n)_0$	$E(n)_1$
.5	.20	.274	.282	13.1(93)	11.7(65)	13.5(54)	13.6(102)	9.1	8.7
	.15	.218	.214	18.9(87)	17.6(56)	19.8(44)	18.9(97)	14.4	13.0
	.10	.196	.160	27.6(71)	25.3(45)	27.1(32)	27.3(90)	21.5	19.6
	.05	.118	.098	42.1(82)	43.9(34)	53.8(23)	42.5(80)	34.5	32.1
.75	.20	.272	.230	7.2(95)	7.0(87)	8.1(61)	7.7(111)	4.1	4.0
	.15	.178	.204	9.0(119)	9.0(61)	9.9(47)	9.3(144)	5.8	5.4
	.10	.218	.164	11.8(107)	10.7(65)	13.6(44)	12.3(123)	8.0	7.7
	.05	.148	.158	18.0(110)	16.4(58)	14.9(33)	19.2(118)	12.9	11.8
1.0	.20	.210	.226	5.8(112)	5.4(73)	6.2(65)	5.8(120)	2.5	2.4
	.15	.194	.172	6.6(119)	6.2(73)	7.0(51)	6.9(162)	3.4	3.2
	.10	.186	.160	7.7(105)	7.3(70)	9.0(61)	8.4(138)	4.5	4.4
	.05	.160	.112	10.1(127)	8.7(70)	10.9(41)	10.6(149)	6.4	5.9
1.5	.20	.138	.182	5.1(98)	5.0(40)	5.3(53)	5.1(108)	1.4	1.4
	.15	.122	.136	5.2(118)	5.1(41)	5.7(48)	5.3(124)	1.7	1.7
	.10	.130	.120	5.6(128)	5.3(53)	6.0(44)	5.7(149)	2.2	2.1
	.05	.098	.092	6.3(168)	5.4(44)	5.9(35)	6.5(181)	3.0	2.7
2.0	.20	.080	.070	5.0(57)	5.0(22)	5.0(18)	5.0(74)	1.1	1.1
	.15	.064	.072	5.0(80)	5.0(23)	5.1(21)	5.1(97)	1.2	1.2
	.10	.050	.066	5.1(101)	5.0(19)	5.3(24)	5.1(120)	1.5	1.4
	.05	.092	.062	5.2(122)	5.1(39)	5.5(21)	5.4(148)	1.7	1.7
3.0	.20	.030	.022	5.0(17)	5.0(4)	5.0(6)	5.0(23)	1.0	1.0
	.15	.018	.018	5.0(26)	5.0(2)	5.0(7)	5.0(37)	1.0	1.0
	.10	.026	.016	5.0(45)	5.0(7)	5.0(7)	5.0(51)	1.1	1.1
	.05	.034	.026	5.0(53)	5.0(13)	5.2(13)	5.0(68)	1.2	1.1

Appendix D

Power

Tables are based upon two hundred Monte Carlo repetitions and are truncated at $2 \cdot E(n)$.

Table D-I-1

Power, Test One

$\theta_0=1$, $\theta_1=1.5$ Minimum Sample Size=5 Asymptotic θ Statistic

Risks	θ_{IN}	Power|k= .5	.75	1.0	1.5	2.0	3.0
.20	.1	.000	.000	.000	.000	.000	.000
.10		.000	.000	.000	.000	.000	.000
.20	.2	.000	.000	.000	.000	.000	.000
.10		.000	.000	.000	.000	.000	.000
.20	.4	.010	.000	.000	.000	.000	.000
.10		.015	.000	.000	.000	.000	.000
.20	.6	.065	.045	.035	.000	.000	.000
.10		.035	.020	.000	.000	.000	.000
.20	.8	.160	.135	.070	.040	.010	.000
.10		.105	.080	.065	.010	.000	.000
.20	1.0	.350	.300	.260	.180	.130	.020
.10		.315	.255	.235	.160	.085	.045
.20	1.2	.540	.495	.330	.430	.355	.425
.10		.515	.500	.510	.420	.465	.365
.20	1.4	.660	.695	.595	.615	.595	.750
.10		.795	.755	.700	.700	.680	.760
.20	1.6	.845	.780	.780	.820	.830	.905
.10		.855	.875	.835	.875	.860	.900
.20	1.8	.885	.835	.870	.850	.935	.965
.10		.965	.880	.905	.950	.935	.965
.20	2.0	.890	.900	.920	.920	.955	1.000
.10		.955	.960	.945	.960	.965	.995
.20	2.2	.920	.915	.925	.965	.995	1.000
.10		.995	.975	.980	.980	.985	.995
.20	2.4	.940	.925	.980	.980	.990	1.000
.10		.970	1.000	.975	.990	.995	1.000
.20	2.6	.960	.980	.940	.995	.995	1.000
.10		.980	.985	.990	.985	1.000	1.000

Table D-I-2

Power, Test One

$\theta_0=1$, $\theta_1=2$ Minimum Sample Size=5 Asymptotic θ Statistic

		Power|K=					
Risk	θ_{IN}	.5	.75	1.0	1.5	2.0	3.0
.20	.1	.000	.000	.000	.000	.000	.000
.10		.000	.000	.000	.000	.000	.000
.20	.2	.005	.000	.000	.000	.000	.000
.10		.000	.000	.000	.000	.000	.000
.20	.4	.035	.010	.000	.000	.000	.000
.10		.010	.005	.000	.000	.000	.000
.20	.6	.085	.030	.005	.000	.000	.000
.10		.055	.040	.005	.000	.000	.000
.20	.8	.170	.105	.030	.005	.000	.000
.10		.190	.075	.025	.015	.000	.000
.20	1.0	.290	.205	.145	.085	.025	.000
.10		.290	.150	.150	.065	.015	.000
.20	1.2	.340	.295	.250	.175	.175	.050
.10		.365	.280	.265	.210	.080	.040
.20	1.4	.470	.470	.380	.400	.345	.209
.10		.460	.470	.450	.420	.370	.285
.20	1.6	.525	.480	.525	.505	.550	.615
.10		.600	.590	.515	.580	.585	.620
.20	1.8	.665	.635	.630	.670	.760	.840
.10		.690	.645	.705	.730	.730	.880
.20	2.0	.725	.805	.750	.775	.825	.930
.10		.800	.760	.795	.795	.890	.945
.20	2.2	.700	.700	.750	.855	.930	.965
.10		.790	.805	.815	.855	.955	.980
.20	2.4	.785	.790	.860	.905	.930	.990
.10		.860	.840	.885	.915	.945	.985
.20	2.6	.785	.850	.910	.955	.979	1.000
.10		.875	.865	.930	.960	.965	.995

Table D-II-1

Power, Test Two

θ_0=1, θ_1=1.5 Minimum Sample Size=5 Asymptotic G Statistic

		Power|k=					
Risks	θ_{IN}	.5	.75	1.0	1.5	2.0	3.0
.20	.1	.000	.000	.000	.000	.000	.000
.10		.000	.000	.000	.000	.000	.000
.20	.2	.000	.000	.000	.000	.000	.000
.10		.000	.000	.000	.000	.000	.000
.20	.4	.005	.005	.000	.000	.000	.000
.10		.000	.000	.000	.000	.000	.000
.20	.6	.010	.010	.005	.005	.000	.000
.10		.005	.005	.005	.000	.000	.000
.20	.8	.035	.045	.060	.020	.000	.000
.10		.015	.015	.025	.020	.000	.000
.20	1.0	.145	.155	.195	.160	.095	.010
.10		.045	.095	.115	.115	.050	.005
.20	1.2	.315	.270	.390	.370	.370	.240
.10		.345	.250	.295	.310	.300	.375
.20	1.4	.560	.490	.550	.585	.615	.600
.10		.655	.640	.535	.645	.585	.565
.20	1.6	.620	.670	.715	.715	.800	.875
.10		.655	.690	.725	.775	.835	.895
.20	1.8	.725	.760	.870	.890	.810	.965
.10		.800	.850	.850	.905	.810	.950
.20	2.0	.880	.795	.880	.900	.955	.975
.10		.850	.860	.810	.925	.955	.985
.20	2.2	.860	.885	.850	.975	.990	.990
.10		.860	.945	.945	.985	.980	1.000
.20	2.4	.825	.895	.950	.950	.990	1.000
.10		.915	.930	.955	.970	.985	.995
.20	2.6	.870	.930	.965	.985	.995	1.000
.10		.900	.910	.975	.990	1.000	1.000

Table D-II-2

Power, Test Two

$\theta_0=1$, $\theta_1=2$ Minimum Sample Size=5   Asymptotic G Statistic

Risk	θ_{IN}	Power|K= .5	.75	1.0	1.5	2.0	3.0
.20	.1	.000	.000	.000	.000	.000	.000
.10		.000	.000	.000	.000	.000	.000
.20	.2	.000	.000	.000	.000	.000	.000
.10		.000	.000	.000	.000	.000	.000
.20	.4	.010	.000	.000	.000	.000	.000
.10		.005	.000	.000	.000	.000	.000
.20	.6	.025	.101	.005	.000	.000	.000
.10		.005	.000	.000	.000	.000	.000
.20	.8	.080	.085	.055	.010	.000	.000
.10		.040	.025	.025	.000	.000	.000
.20	1.0	.155	.220	.150	.050	.005	.000
.10		.090	.130	.115	.040	.005	.000
.20	1.2	.220	.335	.280	.180	.075	.000
.10		.180	.190	.170	.130	.065	.000
.20	1.4	.340	.390	.300	.209	.190	.070
.10		.345	.355	.305	.330	.200	.085
.20	1.6	.435	.515	.485	.445	.395	.260
.10		.435	.435	.425	.475	.415	.280
.20	1.8	.540	.610	.690	.650	.600	.570
.10		.580	.565	.675	.605	.685	.535
.20	2.0	.640	.655	.685	.675	.755	.830
.10		.615	.680	.715	.805	.770	.770
.20	2.2	.685	.735	.770	.805	.925	.895
.10		.770	.815	.775	.845	.885	.875
.20	2.4	.750	.815	.850	.910	.930	.980
.10		.810	.810	.855	.900	.940	.965
.20	2.6	.771	.825	.875	.900	.965	.985
.10		.840	.855	.875	.935	.920	.975

Table D-III-1

Power, Test Three

$\theta_0=1$, $\theta_1=1.5$ Minimum Sample Size=5 Exact G Statistic

Risk	θ_{IN}	Power|K= .5	.75	1.0	1.5	2.0	3.0
.20	.1	.000	.000	.000	.000	.000	.000
.10		.000	.000	.000	.000	.000	.000
.20	.2	.000	.000	.000	.000	.000	.000
.10		.000	.000	.000	.000	.000	.000
.20	.4	.005	.005	.000	.000	.000	.000
.10		.000	.005	.000	.000	.000	.000
.20	.6	.025	.030	.015	.005	.000	.000
.10		.010	.015	.005	.000	.000	.000
.20	.8	.100	.100	.065	.050	.010	.000
.10		.045	.020	.025	.035	.010	.000
.20	1.0	.315	.265	.320	.125	.185	.095
.10		.160	.145	.175	.210	.155	.110
.20	1.2	.480	.435	.535	.445	.520	.480
.10		.520	.525	.450	.465	.515	.480
.20	1.4	.685	.625	.630	.660	.705	.800
.10		.765	.715	.750	.720	.780	.805
.20	1.6	.765	.805	.740	.820	.820	.955
.10		.960	.885	.870	.905	.910	.930
.20	1.8	.885	.880	.850	.920	.935	1.00
.10		.980	.965	.940	.945	.955	.975
.20	2.0	.885	.905	.925	.945	.980	.995
.10		1.000	.975	.975	.965	.995	1.000
.20	2.2	.980	.895	.955	.970	.985	1.000
.10		.990	.995	.980	.995	1.000	1.000
.20	2.4	.960	.955	.950	.995	.990	1.000
.10		.990	.985	.990	.995	.995	1.000
.20	2.6	.980	.965	.975	1.000	.995	1.000
.10		.990	1.000	.990	1.000	.995	1.000

Table D-III-2

<u>Power</u>, <u>Test</u> <u>Three</u>

θ_0=1, θ_1=2.0 Minimum Sample Size=5 Exact G Statistic

		Power|K=					
Risk	θ_{IN}	.5	.75	1.0	1.5	2.0	3.0
.20	.1	.000	.000	.000	.000	.000	.000
.10		.000	.000	.000	.000	.000	.000
.20	.2	.000	.000	.000	.000	.000	.000
.10		.000	.000	.000	.000	.000	.000
.20	.4	.005	.075	.000	.000	.000	.000
.10		.005	.000	.000	.000	.000	.000
.20	.6	.060	.040	.020	.000	.000	.000
.10		.030	.020	.010	.000	.000	.000
.20	.8	.120	.160	.110	.030	.000	.000
.10		.090	.080	.060	.020	.010	.000
.20	1.0	.240	.240	.210	.140	.070	.025
.10		.175	.195	.215	.130	.050	.045
.20	1.2	.370	.375	.340	.315	.340	.215
.10		.265	.335	.280	.345	.325	.275
.20	1.4	.550	.530	.495	.540	.510	.615
.10		.475	.485	.430	.520	.565	.600
.20	1.6	.610	.600	.595	.690	.780	.875
.10		.610	.565	.660	.700	.785	.860
.20	1.8	.680	.675	.750	.815	.830	.955
.10		.810	.730	.730	.785	.890	.960
.20	2.0	.740	.725	.780	.815	.915	.980
.10		.850	.865	.875	.880	.895	1.000
.20	2.2	.795	.825	.815	.915	.950	.990
.10		.810	.900	.865	.920	.980	.995
.20	2.4	.800	.800	.895	.960	.990	1.000
.10		.880	.900	.945	.965	.995	1.000
.20	2.6	.885	.845	.830	.955	.990	1.000
.10		.950	.925	.940	.975	.985	1.000

Appendix E

Comparison of Estimates $\hat{\theta}$, $\hat{K}$, $\hat{G}$ with Actual θ, K, G

Table E-I

Comparison of Estimates $\hat{\theta}$, $\hat{K}$, $\hat{G}$ with Actual θ, K, G when $\theta = 1.0$

Risk	θ	$\hat{\theta}$	K	$\hat{K}$	G	$\hat{G}$	ASN
.20	1.0	1.16	.5	.71	1.0	1.11	17.0
.15	1.0	1.11	.5	.70	1.0	1.08	23.2
.10	1.0	1.06	.5	.67	1.0	1.04	33.1
.05	1.0	1.03	.5	.62	1.0	1.02	52.2
.20	1.0	.98	.75	1.06	1.0	.98	9.8
.15	1.0	1.00	.75	1.06	1.0	1.00	11.9
.10	1.0	1.03	.75	1.03	1.0	1.03	16.7
.05	1.0	.93	.75	.96	1.0	.93	22.4
.20	1.0	.95	1.0	1.43	1.0	.93	7.4
.15	1.0	.96	1.0	1.43	1.0	.94	8.6
.10	1.0	.96	1.0	1.36	1.0	.95	11.3
.05	1.0	.94	1.0	1.37	1.0	.95	15.0
.20	1.0	.95	1.5	2.14	1.0	.92	5.9
.15	1.0	.97	1.5	2.09	1.0	.95	6.4
.10	1.0	.95	1.5	2.21	1.0	.89	6.8
.05	1.0	.92	1.5	2.10	1.0	.84	8.1
.20	1.0	.97	2.0	2.93	1.0	.91	5.4
.15	1.0	.96	2.0	2.88	1.0	.89	5.6
.10	1.0	.95	2.0	2.81	1.0	.87	5.7
.05	1.0	.95	2.0	2.77	1.0	.87	6.6
.20	1.0	.98	3.0	4.29	1.0	.92	5.1
.15	1.0	.99	3.0	4.29	1.0	.96	5.1
.10	1.0	.98	3.0	4.09	1.0	.92	5.2
.05	1.0	.99	3.0	4.22	1.0	.96	5.4

Table E-II

Comparison of Estimates $\hat{\theta},\hat{k},\hat{G}$ with Real $\hat{\theta},\hat{k},\hat{G}$ when θ=1.5

Input Risk	θ	$\hat{\theta}$	k	$\hat{k}$	G	$\hat{G}$	ASN
.20	1.5	2.12	.5	.66	1.22	1.64	46.7
.15	1.5	1.96	.5	.64	1.22	1.54	63.4
.10	1.5	1.99	.5	.58	1.22	1.49	102.2
.05	1.5	1.86	.5	.57	1.22	1.42	145.0
.20	1.5	1.79	.75	1.06	1.35	1.85	21.5
.15	1.5	1.86	.75	1.01	1.35	1.87	30.1
.10	1.5	1.80	.75	1.02	1.35	1.82	42.3
.05	1.5	1.78	.75	.95	1.35	1.73	67.1
.20	1.5	1.69	1.0	1.41	1.5	2.10	15.0
.15	1.5	1.78	1.0	1.41	1.5	2.25	17.8
.10	1.5	1.72	1.0	1.36	1.5	2.34	25.6
.05	1.5	1.68	1.0	1.29	1.5	1.95	37.6
.20	1.5	1.58	1.5	2.22	1.84	2.76	9.1
.15	1.5	1.59	1.5	2.12	1.84	2.67	10.3
.10	1.5	1.58	1.5	2.18	1.84	2.71	14.1
.05	1.5	1.61	1.5	2.18	1.84	2.82	18.9
.20	1.5	1.55	2.0	2.89	2.25	3.55	6.7
.15	1.5	1.53	2.0	2.94	2.25	3.49	8.0
.10	1.5	1.59	2.0	2.79	2.25	3.65	9.1
.05	1.5	1.57	2.0	3.03	2.25	3.92	12.0
.20	1.5	1.49	2.0	4.52	3.38	6.06	5.7
.15	1.5	1.51	3.0	4.34	3.38	6.55	6.1
.10	1.5	1.52	3.0	4.45	3.38	6.44	6.5
.05	1.5	1.53	3.0	4.56	3.38	6.95	7.3

Table E-III

Comparison of Estimates $\hat{\theta}$, $\hat{K}$, $\hat{G}$ with Actual θ, F, G when θ=2.0

Risk	θ	$\hat{\theta}$	K	$\hat{K}$	G	$\hat{G}$	ASN
.20	2.0	2.99	.5	.71	1.41	2.18	17.8
.15	2.0	3.02	.5	.70	1.41	2.17	23.8
.10	2.0	2.72	.5	.67	1.41	1.96	35.0
.05	2.0	2.73	.5	.62	1.41	1.86	52.9
.20	2.0	2.48	.75	1.07	1.68	2.64	10.0
.15	2.0	2.40	.75	1.06	1.68	2.53	12.6
.10	2.0	2.48	.75	1.06	1.68	2.62	17.5
.05	2.0	2.39	.75	1.02	1.68	2.43	24.2
.20	2.0	2.21	1.0	1.45	2.00	3.16	7.9
.15	2.0	2.24	1.0	1.48	2.00	3.30	9.2
.10	2.0	2.17	1.0	1.44	2.00	3.05	11.5
.05	2.0	2.24	1.0	1.41	2.00	3.12	15.4
.20	2.0	2.09	1.5	2.11	2.83	4.74	6.1
.15	2.0	2.06	1.5	2.12	2.83	4.63	6.6
.10	2.0	2.11	1.5	2.13	2.83	4.91	7.4
.05	2.0	2.11	1.5	2.26	2.83	5.41	8.9
.20	2.0	2.02	2.0	2.90	4.0	7.68	5.5
.15	2.0	2.01	2.0	2.95	4.0	7.84	5.8
.10	2.0	2.04	2.0	2.94	4.0	8.13	6.1
.05	2.0	2.09	2.0	2.96	4.0	8.86	6.7
.20	2.0	1.98	3.0	4.16	8.0	17.14	5.1
.15	2.0	1.96	3.0	4.38	8.0	19.06	5.1
.10	2.0	1.99	3.0	4.43	8.0	21.08	5.2
.05	2.0	2.00	3.0	4.29	8.0	19.56	5.4

Vita

Richard L. Hoffert was born on 28 December 1940 in Caro, Michigan. He graduated from high school in Fremont, Ohio in 1958 and attended the United States Air Force Academy from which he received the degree of Bachelor of Science in Public Policy in 1962. Upon graduation he was commisioned as a 2nd Lieutenant in the USAF. He attended pilot training and served in a variety of flying and logistics assignments before entering the Air Force Institute of Technology in June 1975.

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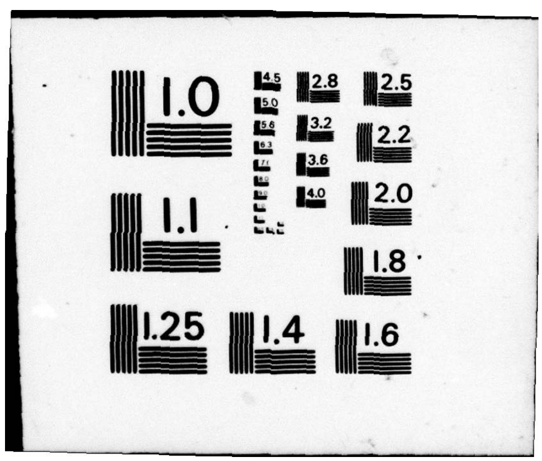
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theta

Three Sequential Likelihood Ratio Tests were constructed to choose between two values of the scale parameter (θ) of a Weibull distribution with a location parameter of zero, an unknown shape parameter (K) replaced by its maximum likelihood estimate, and error bounds of 0.05, 0.10, 0.15, and 0.20. → next page

The null hypothesis for all tests was θ equals 1.0. Two alternate hypotheses were used: θ equals 1.5 and 2.0.

The first two tests were based upon a general procedure for sequential tests for a parameter in the presence of a nuisance parameter by D. R. Cox in Sankhya A, Vol. 25. This method replaces the likelihood ratio with an asymptotically equivalent test statistic.

The tests were then truncated and the effects of this truncation upon the error bounds were studied. All tests were conducted with six input values of K ranging from .5 to 3.0. Newton-Raphson procedures were used to estimate K. The entire test was conducted using Monte Carlo procedures. The power of the truncated tests was also investigated.

Tabulated output data provides the output Type One and Type Two errors and the average sample numbers.